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CONHECIMENTO PEDAGÓGICO DO CONTEÚDO PARA O ENSINO DE NÚMEROS E OPERAÇÕES: UM ESTUDO COM FUTUROS PROFESSORES

PEDAGOGICAL CONTENT KNOWLEDGE FOR TEACHING NUMBERS AND OPERATIONS: A STUDY WITH PROSPECTIVE TEACHERS

CONOCIMIENTO PEDAGÓGICO DEL CONTENIDO PARA LA ENSEÑANZA DE NÚMEROS Y OPERACIONES: UN ESTUDIO CON FUTUROS PROFESORES

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RESUMO

Introdução: A Formação Inicial de Professores (FIP) tem como missão preparar os futuros professores para a docência, visando dotá-los das competências e conhecimentos científicos, técnicos e pedagógicos de base para o desempenho profissional da prática docente, em particular do conteúdo e conhecimento pedagógico do conteúdo (*PCK*).
Objetivo: Compreender o *PCK* necessário para o ensino de Números e Operações dos futuros professores em formação inicial.
Métodos: Foi adotada uma metodologia quantitativa, utilizando um questionário para recolher dados de 136 futuros professores em FIP de quatro instituições públicas de Ensino Superior em Angola.
Resultados: Verificam-se variações significativas nas perceções dos participantes em relação às estratégias pedagógicas para o ensino de Números e Operações, com maior destaque para desafios associados com a resolução de problemas e aplicação de cálculo com números inteiros e decimais. Foram identificadas diferenças estatisticamente significativas entre as instituições, evidenciando desigualdades na formação recebida sobre o *PCK*. A análise das correlações mostrou interdependência entre as dimensões avaliadas.
Conclusão: O estudo reforça a necessidade de intervenções formativas especificas, que integrem estratégias pedagógicas inovadoras e contextuais, para melhorar o desenvolvimento do *PCK* durante a formação inicial. Contribui para a literatura ao explorar lacunas formativas num contexto pouco investigado, oferecendo subsídios para a melhoria do ensino de Matemática na FIP em Angola.

Palavras-chave: conhecimento pedagógico do conteúdo; números e operações; formação inicial de professores

ABSTRACT

Introduction: The mission of Initial Teacher Education (ITE) is to prepare future teachers for teaching with the basic scientific, technical, and pedagogical skills and knowledge for the professional performance of teaching practice, in particular content and pedagogical content knowledge (PCK).
Objective: To understand the PCK needed to teach Numbers and Operations to future teachers in initial training.
Methods: A quantitative methodology was adopted, using a questionnaire to collect data from 136 future teachers in training from four public higher education institutions in Angola.
Results: There are significant variations in participants' perceptions of pedagogical strategies for teaching Numbers and Operations, with greater emphasis on challenges associated with problem-solving and the application of calculation with whole numbers and decimals. Statistically significant differences were identified between the institutions, indicating inequalities in the training received on Pedagogical Content Knowledge (PCK). Analyzing the correlations revealed an interdependence between the assessed dimensions.
Conclusion: The study reinforces the need for specific training interventions that integrate innovative and contextually relevant pedagogical strategies to enhance the development of Pedagogical Content Knowledge (PCK) during initial training. It contributes to the literature by exploring training gaps in a context that has been little investigated, offering insights for improving mathematics teaching in Initial Teacher Education (ITE) in Angola.

Keywords: pedagogical content knowledge; numbers and operations; initial teacher training

RESUMEN

Introducción: La misión de la Formación Inicial del Profesorado (FIP) es preparar a los futuros profesores para la docencia, con el objetivo de dotarles de las competencias y conocimientos científicos, técnicos y pedagógicos básicos para el desempeño profesional de la práctica docente, en particular los conocimientos sobre contenidos y contenidos pedagógicos (*PCK*).
Objetivo: Comprender el *PCK* necesario para enseñar Números y Operaciones a futuros profesores en formación inicial.
Métodos: Se adoptó una metodología cuantitativa, utilizando un cuestionario para recoger datos de 136 futuros profesores en formación de cuatro instituciones públicas de enseñanza superior de Angola.
Resultados: Muestran variaciones significativas en las percepciones de los participantes sobre las estrategias pedagógicas para la enseñanza de Números y Operaciones, con mayor énfasis en los desafíos asociados a la resolución de problemas y a la aplicación del cálculo con números enteros y decimales. Se identificaron diferencias estadísticamente significativas entre las instituciones, mostrando desigualdades en la formación recibida sobre PCK. El análisis de las correlaciones mostró interdependencia entre las dimensiones evaluadas.
Conclusión: El estudio refuerza la necesidad de intervenciones formativas específicas que integren estrategias pedagógicas innovadoras y contextuales para mejorar el desarrollo de las PCK durante la formación inicial. Contribuye a la literatura explorando las lagunas formativas en un contexto poco investigado, ofreciendo subsidios para mejorar la enseñanza de las matemáticas en la FIP en Angola.

Palabras Clave: conocimiento pedagógico del contenido; números y operaciones; formación inicial del profesorado

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INTRODUCTION

Initial Teacher Education (ITE) is essential for ensuring the quality of education, particularly in countries where pedagogical challenges persist, such as Angola. ITE should articulate content knowledge (CK) and Pedagogical Content Knowledge (PCK) in these countries, preparing future teachers to teach effectively. Nevertheless, CK is not enough to help students achieve meaningful learning. It is essential to deepen the understanding of Pedagogical Content Knowledge (PCK) so that future teachers feel better prepared to teach the topic of Numbers and Operations (Martins et al., 2024). Despite being one of the core contents of the ITE, prospective teachers' difficulties in choosing effective methodologies persist. The effectiveness of these strategies and prospective teachers' perceptions of their application are little researched, especially in countries like Angola, where there is a limited number of studies on Pedagogical Content Knowledge (PCK) for teaching Numbers and Operations (Joaquim et al., 2023; Joaquim et al., 2024). This study aims to fill this gap by analysing future teachers' perceptions of applying pedagogical strategies for teaching numbers and operations.

Recent studies, such as those by Martins et al. (2021), Scheiner et al. (2023), and Joaquim et al. (2024), have demonstrated that PCK enables the identification of students' difficulties and the design of targeted interventions to enhance teaching practices. Teachers' positive beliefs support the learning of numbers and operations. However, problems still need to be investigated, as gaps in initial teacher education have been identified, highlighting the need for strategies that integrate PCK into classroom practices (Piñeiro & Calle, 2023). This study aims to investigate the development of PCK in numbers and operations to enhance the quality of mathematics teaching in ITE settings for primary education, strengthen pedagogical capacity, and promote mathematical thinking.

These results highlight critical gaps in preparing future primary school teachers, particularly in terms of PCK for teaching numbers and operations during ITE. To better understand and address these gaps, this study is guided by two research questions: Q1: How do future primary school teachers in Angola perceive the pedagogical strategies for teaching numbers and operations they learned during their training? Q2: What difficulties and training needs do future primary school teachers identify regarding the PCK for teaching numbers and operations? Understanding these perceptions and challenges is essential for contextualizing the Professional Capital (PCK) for teaching numbers and operations in Initial Teacher Education (ITE) and identifying key areas for improvement that align with the needs of future teachers.

1. THEORETICAL FRAMEWORK

1.1. Mathematical knowledge for teaching and learning mathematics

Teaching mathematics in ITE requires the integration of CK and PCK, which are essential for preparing future teachers to teach mathematics effectively (Shulman, 1986; Ball et al., 2008; da Ponte, 2023). In the context of ITE for primary education, particularly mathematics education, CK refers to understanding mathematical concepts and procedures. At the same time, PCK includes pedagogical strategies, student understanding, and curricular articulation (Copur-Gencturk & Tolar, 2022). In the ITE, the development of mathematical knowledge for teaching (MKT) promotes essential and reflective pedagogical practices that meet the challenges of the teaching profession (Martins et al., 2021). For future teachers, MKT should consist of CK and PCK, both essential domains for developing effective mathematics teaching (Scheiner et al., 2023). Therefore, the theoretical framework of this study is based on the contributions of Shulman (1986) and Ball et al. (2008), which integrate the perspectives of PCK and MKT. Future students must clearly understand the PCK for teaching mathematics in primary education, as this knowledge is fundamental to promoting good practice and ensuring more effective initial teacher education (Copur-Gencturk & Li, 2023). In this sense, it is necessary to implement training strategies that facilitate access to retraining programs. Retraining may include courses designed to overcome mathematics difficulties and out-of-school training to continually improve future teachers' understanding of content knowledge and pedagogical practices (Rice et al., 2024). On the other hand, these courses help identify gaps in future teachers' knowledge that need improvement. These courses are also essential as they help improve prospective teachers' mathematical knowledge acquired during initial training, thereby contributing to the development of the knowledge required for teaching practice (Galvão & Ponte, 2018). There is a need to explore prospective teachers' teaching practices in numbers and operations from initial training onwards. However, analyzing the PCK helps to gain a deeper understanding of these practices.

1.2. Pedagogical content knowledge for teaching Numbers and Operations

The ITE should explore the PCK sub-domains of knowledge of content and teaching, knowledge of content and students (KCS), and knowledge of content and curriculum in numbers and operations (Ball et al., 2008). This topic requires preservice teachers to understand students' thinking, anticipate difficulties, and develop effective teaching strategies (Copur-Genctuk & Li, 2023). Developing a sound PCK enables future teachers to identify errors, develop effective interventions, and promote mathematical thinking to overcome learning difficulties (Fukaya & Uesaka, 2023; Ko et al., 2024). On the other hand, it is necessary to deepen the PCKNO, and the ITE should include innovative practices such as problem-based learning, integration of practical experience, and pedagogical reflection (Galvão & Ponte, 2018; Martin & Jamieson-Proctor, 2022; Pascual Martin et al., 2023). These elements

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are essential for developing the PCKNO of future teachers and ensuring effective mathematics teaching in primary education. In this context, the dimensions, categories of analysis, and operational definitions that help to understand the PCKNO required for teaching practice in primary education have been conceptualized (Table 1).

Table 1 – Dimension, categories, and operational definition

Dimension	Categories	Operational setting
Understanding the number sense	Teaching strategies for numerical concepts.	These methods and tools are used to teach concepts such as cardinality, order, and counting and include hands-on activities, manipulatives, and graphical representations.
	Representation and contextualisation of the meaning of number	It refers to using different representations (visual, symbolic, and iconic) and contexts (everyday life, games, and stories) to promote an understanding of numbers.
Calculation with integers and decimal numbers	Teaching strategies for basic arithmetic instruction	It includes teaching methods related to basic operations (addition, subtraction, multiplication, and division) involving manipulatives, problem-solving, and decimal alignment.
	Application of rules and procedures in operations	It focuses on explaining the essential rules and procedures for operations with integers and decimals, including sign rules and decimal alignment, with an emphasis on practical application.
Problem-solving	Teaching strategies for solving problems	It includes methods for identifying key information, developing resolution plans, and verifying answers, encouraging creativity and critical thinking.
	Integrating Technology into Problem Solving	Refers to using technological tools, such as educational software or applications, to promote mathematical problem-solving and develop logical reasoning.

2. METHODS

The study employed a quantitative approach to systematically analyze responses regarding prospective teachers' perceptions of the PCKNO. The selection of this approach enables the identification of patterns, trends, and training gaps, thereby contributing to a deeper understanding of the current state of PCK for teaching Numbers and Operations in the Angolan context (Cohen et al., 2018). The study design is descriptive and cross-sectional (Cohen et al., 2018), utilizing a single data collection point. It enables the mapping of participants' perceptions and an understanding of the nature of PCK for teaching Numbers and Operations acquired during the ITE (León & Monteiro, 2020). Studies of this nature may prepare future research focusing on more practical longitudinal interventions.

2.1 Sample

One hundred thirty-six (136) students in ITE for primary education participated in this study from the 2nd, 3rd, and 4th years of degree programs at four public higher education institutions in Angola. ITE programs guided the selection of institutions with an emphasis on primary education. Non-probability convenience sampling was chosen due to the study's accessibility, availability, and logistical considerations. Although this type of sampling does not allow for statistical generalizations for the entire population, it is widely recognized in the literature as suitable for exploratory studies in specific teaching contexts, where the main objective is to understand perceptions, difficulties, and needs regarding training (Etikan, Musa, & Alkassim, 2016). This approach enabled the collection of significant data on the reality of future teachers in Initial Teacher Education (ITE), aligning with the study's objectives.

2.2 Data collection instruments

Data were collected using a questionnaire comprising 28 multiple-choice questions, and our analysis focused on 23 of these, which pertained to understanding number sense, calculation with integers and decimal numbers, and problem-solving dimensions. The first group of questions collected the socio-demographic data of future teachers, including their gender, age, academic background, professional status, and length of service if they were already teaching. Next, it was structured in three dimensions: Understanding number sense, calculation with integer and decimal numbers, and problem-solving. This instrument assessed prospective teachers' perceptions of PCK in the area of Numbers and Operations (Shulman, 1986; Ball et al., 2008; León & Montero, 2020). Each of the 23 questions was designed to reflect PCK in the context of teaching numbers and operations in elementary school (students aged 5-11). The questions aimed to assess prospective teachers' perceptions of factual PCK and their ability to select appropriate pedagogical strategies based on didactic principles. Each question presents an instructional context, followed by four to five mutually exclusive response options. The options reflect different levels of PCK development, including elements related to content delivery, pedagogical suggestions, developing mathematical thinking, the use of manipulatives, multiple representations, and problem-solving (Table 2). The questionnaire was designed for this study and validated by experts, including two professors of Mathematics (I and II), two professors of Didactics of Mathematics, and a Professor of Statistics. The

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experts are professors from a University in Northern Portugal and two Universities in Angola. Data was collected face-to-face in sessions organized by the participating institutions, respecting all ethical procedures, with the authorization of the ethics committee registered under Doc. 32-CE-UTAD-2024.

Table 2 – Examples of the questions

Dimension	Question example
Understanding number sense	What teaching tasks or resources would you use to help elementary school pupils understand the concept of cardinality? a) mathematical games to practice counting; b) lectures on the concept of cardinality; c) paper and pencil exercises only; d) stories to illustrate the concept of cardinality; e) other, which?
Calculation with integers and decimal numbers	What would be an appropriate approach to teaching the addition of whole numbers? a) teach the sign rule technique; b) show examples of adding integers with different signs; c) start the lesson by adding positive integers; d) not dealing with adding whole numbers and focusing only on decimals.
Problem-solving	What would be an appropriate approach to teaching the addition of whole numbers? a) teach the sign rule technique; b) show examples of adding integers with different signs; c) start the lesson by adding positive integers; d) not dealing with adding whole numbers and focusing only on decimals.

2.3 Statistical analysis

The data were analyzed quantitatively using SPSS *v.*29. Since the sample variables did not follow a normal distribution, both descriptive and inferential statistics, as well as correlation and significance tests, were employed to compare groups and identify variability in the analysis. The analysis was structured according to the dimensions of the PCK (Tables 1 and 2), allowing for the identification of patterns and gaps, as well as highlighting critical areas for potential training interventions. For data analysis, 23 multiple-choice questions were chosen from the 28 items in the questionnaire. The remaining five items, which were qualitative and reflective, were omitted from the quantitative analysis. The questions examined correspond to three dimensions of PCK: six questions related to understanding number sense, twelve about calculations with whole numbers and decimals, and five on problem-solving. Each question scored 0.5 or 1 point, depending on its structure and cognitive demand. The scores were standardized on a scale of 0 to 20 to ensure comparability between the dimensions (Table 2). This procedure allowed for an interpretation of the results in a balanced manner across dimensions with varying numbers of questions and relative weights. Table 3 presents the points and grades, which have been normalized, ensuring consistency in the reading and analysis of the data.

Table 3 – Student 1 and 136 grades for the questions of the dimension 'Understanding number sense'

	Understanding number sense (Q: Question)						Points	Grade (0-20)
Student	Q1	Q2	Q3	Q4	Q5	Q6		
1	1	0	1	0	1	0	3 of 6	10.00
				…				
136	1	1	0	1	0	1	4 of 6	13.33

3. RESULTS

The results are presented according to the dimensions and categories of analysis (Table 1), enabling the practical analysis of the collected data, which may reflect essential aspects of the PCK for teaching numbers and operations. Thus, the dimension of understanding number sense (Dimension I) includes the following categories to be analyzed: teaching strategies for numerical concepts and representation and contextualization of the meaning of numbers, both designed to answer Q1. The dimension of calculation with integer and decimal numbers (Dimension II) comprises the following categories: teaching strategies for basic arithmetic instruction and applying rules and procedures in operations, which aim to understand Q1. Finally, the dimension of Problem-solving (Dimension III) comprises the following categories: teaching strategies for solving problems and integrating Technology into problem-solving, both of which seek to address elements related to Q1 and Q2.

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3.1. Descriptive Statistics Results

The descriptive analysis focused on the mean, standard deviation, minimum, and maximum values of the observations recorded in the institutions participating in the study and is presented in Table 4. These values align with the maximum values shown in Table 3 as examples.

Table 4 - Results of descriptive statistics of the 136 future teachers' grades (0-20)

Dimension	Mean	Standard Deviation	Minimum	Maximum	c.v. (%)
Understanding number sense	8.18	4.32	0	16.7	41.78
Calculation with integers and decimal numbers	7.02	3.68	0	15.0	52.41
Problem-solving	9.30	6.25	0	20.0	67.24

According to the results in Table 4 and the dimension 'Understanding number sense,' the minimum value recorded was 0, while the maximum value was 16.7, suggesting that some participants have low or high response rates. For this dimension, the mean was 8.18, with a standard deviation of 4.32, indicating average and consistent perceptions of pedagogical strategies for developing teaching skills in this dimension. In the dimension 'Calculation with integer and decimal numbers' results, the minimum value was 0, while the maximum value was 15 (the most diverse perceptions in this dimension). The mean was 7.02, with a standard deviation of 3.68. In the 'Problem-solving' dimension, the minimum value was 0, while the maximum value was 20.0. The mean was 9.30, and the standard deviation was 6.25. Although the 136 answers exhibited high variability in all three dimensions, the greatest variability was observed in the 'Problem-solving' dimension (67.24%), indicating a challenge in future teachers' understanding of the PCK for teaching numbers and operations in this dimension.

Table 5 presents the results of this analysis, highlighting how future teachers from each institution perceived the pedagogical strategies related to teaching numbers and operations. The average Kruskal-Wallis ranks for each institution's assessed dimensions were calculated to analyze the differences in perceptions of future teachers from the four higher education institutions (referred to as U1, U2, U3, and U4).

Table 5 - Ranking of average posts by institution using the Kruskal-Wallis test

Dimension	Institution codes	N	Rank
Understand number sense	*U1*	*46*	*56.67*
	U2	*25*	*62.70*
	U3	*43*	*73.27*
	U4	*22*	*90.50*
	Total	*136*	
Calculation with integers and decimals	*U1*	*46*	*66.09*
	U2	*25*	*63.34*
	U3	*43*	*62.34*
	U4	*22*	*91.45*
	Total	*136*	
Problem-solving	*U1*	*46*	*57.77*
	U2	*25*	*59.56*
	U3	*43*	*76.63*
	U4	*22*	*85.20*
	Total	*136*	

According to the results in Table 5, Institution U4 (90.5) had the highest rank in the dimension 'Understanding number sense,' indicating very positive perceptions, followed by U3 (73.27). Institutions U1 (56.67) and U2 (62.70) obtained ranks, suggesting the existence of specific challenges in this dimension, possibly related to the difficulties future teachers face in developing PCK and number sense. In the dimension 'Calculation with integers and decimal numbers,' U4 (91.45) again stood out, followed by U1 (66.09), while U2 (63.34) and U3 (62.34) had lower mean scores. U2 and U3 have more moderate perceptions, illustrating challenges in this dimension. Regarding 'Problem-solving,' U4 (85.20) stood out, followed by U3 (76.63). Institutions U1 (57.77) and U2 (59.56) had the lowest ranks, indicating difficulties in this dimension. Institution U4 consistently has the highest ranks in all dimensions, followed by Institution U3, except in 'Calculation with integers and decimal numbers,' where they are lower. At the same time, Institutions U1 and U2 had less favorable perceptions, suggesting a need to improve their teaching practices in the topics with lower ranks.

3.2. Results of the statistical significance test

To determine if there were significant differences in the perceptions of future teachers regarding the PCK for teaching numbers and operations adopted by each institution, a comparative analysis was conducted of the ranks obtained by each institution in the

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assessed dimensions. The Kruskal-Wallis test was used (a = 0.05, a 5% significance level). In this analysis, the H value refers to the test statistic that measures the differences between the groups, the degrees of freedom (df) refers to the number of groups compared, and the p-value (Sig.) determines the probability that these differences occur by chance or whether they are statistically significant. The results of this statistical analysis are presented in Table 6.

Table 6 – Kruskal-Wallis test for each of the three dimensions, using Institutions as the grouping variable

	Understanding number sense	Calculation with integers and decimal numbers	Problem-solving
Kruskal-Wallis H	*13.195*	*9.406*	*11.001*
df	*3*	*3*	*3*
Sig.	*.004*	*.024*	*.012*
Eta squared (size effect)	*.077*	*.048*	*0.061*

The dimension 'Understanding number sense' showed significant differences in participants' perceptions between the institutions (H = 13.195, df = 3, p = 0.004 < 0.05). The dimension 'Calculation with integer and decimal numbers' also showed significant variations (H = 9.406, df = 3, p = 0.024 < 0.05 = a). Similarly, the dimension 'Problem-solving' showed significant statistical differences between the institutions (H = 11.001, df = 3, p = 0.012 < 0.05 = a). In summary, all the results show statistically significant differences in participants' perceptions in all assessed dimensions in the PKC for teaching numbers and operations. Those statistically significant differences may indicate the existence of disparities in training practices that reflect inequalities or curricular particularities in the training of future teachers and the profile of students entering the ITE course. The computation of eta squared (Tomczak & Tomczak, 2014) in Table 6 shows that only a medium relationship between variables was found (Cohen et al., 2018, p. 746). In this way, they may highlight the need to investigate the institutional factors that influence these differences. Therefore, strengths and areas for improvement in the ITE regarding PCK can be identified. The pairwise comparisons from SPSS v.29 were used to determine which pairs of institutions showed significant differences (a = 0.05, a 5% significance level, Table 7).

Table 7 – Pairwise comparisons following the Kruskal-Wallis test

Dimension	Pair of Groups	p-value	Statistical significant
Understand number sense	*U1 vs. U2*	*.522*	*No*
	U1 vs. U3	*.039*	*Yes*
	U1 vs. U4	*.001*	*Yes*
	U2 vs. U3	*.267*	*No*
	U2 vs. U4	*.012*	*Yes*
	U3 vs. U4	*.082*	*No*
Calculation with integers and decimal numbers	*U1 vs. U2*	*.918*	*No*
	U1 vs. U3	*.649*	*No*
	U1 vs. U4	*.004*	*Yes*
	U2 vs. U3	*.776*	*No*
	U2 vs. U4	*.013*	*Yes*
	U3 vs. U4	*.012*	*Yes*
Problem-solving	*U1 vs. U2*	*.852*	*No*
	U1 vs. U3	*.021*	*Yes*
	U1 vs. U4	*.006*	*Yes*
	U2 vs. U3	*.078*	*No*
	U2 vs. U4	*.023*	*Yes*
	U3 vs. U4	*.395*	*No*

In the 'understanding number sense' dimension, U4 performed significantly better than U1 (p = 0.001< 0.05) and U2 (p = 0.012< 0.05). U3 also performed better than U1 (p = 0.039< 0.05). However, the remaining comparisons (U1 *vs.* U2, U2 *vs.* U3, and U3 *vs.* U4) revealed no statistically significant differences. Nevertheless, the latter pair is close to the conventional significance threshold, suggesting a possible trend. In the "calculation with integers and decimals" dimension, future teachers from institution U4 demonstrated significantly higher performance levels than those from institutions U1 (p = 0.004< 0.05), U2 (p = 0.013< 0.05), and U3 (p = 0.012< 0.05). However, the remaining comparisons (U1 *vs*. U2, U1 *vs*. U3, and U2 *vs*. U3) showed no statistically significant differences (p = 0.918, p = 0.649, and p = 0.776, respectively). In terms of the problem-solving dimension, U3 and U4 held significantly higher ranks than U1 (p = 0.021 and p = 0.006, respectively), and U4 also excelled compared to U2 (p = 0.023< 0.05). However, there were no significant differences between the following pairs: U1 *vs*. U2 (p = 0.852), U2 *vs*. U3 (p = 0.078), and U3 *vs*. U4 (p = 0.395< 0.05). Overall, the results indicate that Institution U4 outperformed the other institutions across the three dimensions of PCK analyzed, particularly in 'Calculation' and 'Problem-solving.? This result suggests that U4 may have more organized training practices, especially those focused on enhancing specific PCK skills related to teaching numbers and operations.

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Furthermore, these results indicate the need for more effective pedagogical practices at ITE. This result aligns with Martins et al. (2023), who emphasized the importance of contextualized pedagogical strategies in strengthening mathematical thinking and better preparing future teachers. On the other hand, Institution U4 is considered to have the best results in all assessed dimensions, as it achieved significantly more favorable rankings compared to the other three institutions. Those results indicate more effective and successful pedagogical practices in developing prospective teachers' competence in Numbers and operations. According to these comparisons, the results suggest that exploring future teachers' difficulties with PCK would be appropriate. These difficulties would enable the identification of areas that require further research to enhance the knowledge of future teachers. However, it is necessary to establish correlations between the analyzed dimensions to gain a more comprehensive understanding of future teachers' perceptions.

3.3. Results of the correlation test

Spearman's correlation test was used to assess the strength and direction of the relationships between the three dimensions based on the values of the correlation coefficients (CC) and the levels of statistical significance (Sig., compared with a 1% or 5% significance level). This test identified relationships and interdependencies in the development of the PCK for teaching numbers and operations to future primary school teachers. Table 8 presents those results.

Table 8– Spearman's correlation test

Dimension			Understanding number sense	Calculation with integers and decimal numbers	Problem-solving
Spearman's rho	Understanding number sense	CC	*1.000*	*.048*	*.226***
		Sig. (2 tails)		*.581*	*.008*
		N	*136*	*136*	*136*
	Calculation with integers and decimal numbers	CC	*.048*	*1.000*	*.215**
		Sig. (2 tails)	*.581*		*.012*
		N	*136*	*136*	*136*
	Problem-solving	CC	*.226***	*.215**	*1.000*
		Sig. (2 tails)	*.008*	*.012*	
		N	*136*	*136*	*136*

**The correlation is significant at the 0.01 level (2 tails).
*The correlation is significant at the 0.05 level (2 tails).

The results show that the dimension 'understanding number sense' significantly correlates with the dimension 'problem-solving.' 'Understanding number sense' statistically also significantly correlates with the dimension 'problem-solving ' (CC = 0.226 and p = 0.008). On the other hand, 'understanding the number sense' did not show a statistically significant correlation with 'calculation with integer and decimal numbers' (CC = 0.048 and p = 0.581), indicating relative independence between these two dimensions in the analysis context. In the case of 'problem-solving,' the data shows a positive and significant correlation between 'calculation with integers and decimal numbers' (CC = 0.215 and p = 0.012), supporting the hypothesis that effective knowledge of integer and decimal numbers does play an important role in the ability to 'problem-solving.' The lack of a significant correlation between 'understanding the number sense" and 'calculation with integers and decimal numbers' suggests that these skills can be improved during initial training, requiring greater integration of PCK. Copur-Gencturk and Tolar (2022) and Li et al. (2025) reported the same trend. Despite these statistics, the three dimensions studied are essential for improving future teachers' ability to deal with 'Problem-solving' situations.

4. DISCUSSION

The results show significant differences in prospective primary teachers' perceptions of PCK regarding teaching numbers and operations, a key area of primary teachers' mathematical knowledge. The descriptive analyses of these results showed that 'Calculation with integer and decimal numbers' had the highest mean value, indicating that those teachers are more familiar with performing basic mathematical operations (Lo, 2020). However, there is evidence that future teachers face challenges in selecting strategies that facilitate effective teaching. On the other hand, 'Problem-solving' stands out as the dimension that presents consistent challenges, with a lower mean value, reflecting the existence of gaps in initial education that involve the development of integrated and practiced skills.

Comparisons between institutions revealed significant statistical differences in the training of future teachers. In this context, Institution U4 performed higher in 'Problem-solving.' In contrast, Institutions U1 and U2 showed a higher concentration of participants in the medium difficulty levels, i.e., with gaps. These gaps suggest a need for interventions in ITE for primary education, with a focus on enhancing PCK, particularly in the dimensions classified as levels 1 and 2, i.e., understanding number sense and problem-solving, two essential elements of the mathematics curriculum in ITE (Lo, 2020). The statistical significance

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tests confirmed these differences. Statistically significant differences were found between the institutions in all the analyzed dimensions. Specifically, 'Understanding number sense' differences between institutions confirm that the development of PCK varies between them. It was possible to identify specific pairs of institutions with statistically significant differences (although moderate differences based on the size effects, Table 7) in the higher education institutions studied, reinforcing the idea that it is important to include guidance on the use of particular strategies for the development of PCK that helps to understand the concepts of Numbers and operations in ITE (Li et al., 2025).

Finally, the last calculation of Spearman's rho showed a moderate relationship between 'problem-solving' with 'calculation with integers and decimal numbers' (CC = .215) and understanding number sense (CC = .226). The difference between 'Problem-solving' and 'Calculation with integers and decimal numbers' indicates that operation literacy is essential for developing more complex skills related to Numbers and operations. In contrast, the relationship between understanding number sense and calculation with integers and decimal numbers did not show a significant correlation. This result suggests that in the context of teaching 'Numbers and operations,' an improvement in PCK for 'understanding number sense' may be associated with an improved understanding of PCK for 'calculation with integers and decimal numbers.' The findings are consistent with the literature, particularly emphasizing the importance of developing pedagogical content knowledge in initial teacher education (Shulman, 1987; Ball et al., 2008) and its influence on developing effective teaching, such as in the area of 'Number and Operations.'

The results indicate that higher values of Institution U4 in several dimensions reflect what Martins et al. (2023) argue about the importance of contextualized teaching practices in promoting mathematical thinking. For example, the correlations between dimensions, especially 'problem-solving' and 'calculation with integers and decimal numbers,' are supported by studies by Sianturi et al. (2024). They also highlight the interdependence between operations skills and problem-solving. The lack of significant correlations between understanding number sense and computation with whole numbers and decimals suggests, as discussed by Copur-Gencturk and Li (2023), that these competencies can be developed independently in ITE, requiring greater integration of the PCK.

Analyzing the results enables us to answer the two research questions. Regarding Q1, prospective teachers have different perceptions of the three dimensions of PCKNO. Higher performance in calculations involving whole numbers and decimals suggests greater familiarity with procedural methods. In contrast, lower scores in understanding number sense and problem-solving, combined with high variability, suggest a weaker grasp of strategies centered on these areas. This pattern confirms the trend identified by Rowland et al. (2020) that ITE favors procedural approaches over practices that stimulate deep mathematical thinking. As for Q2, the greatest difficulties arise in the problem-solving dimension, which has the lowest average and the highest dispersion. These results may show weaknesses in practices that prioritize reasoning, autonomy, and interpreting mathematical processes. The lack of correlations between the analyzed dimensions reveals a disconnect between the mathematical and didactic knowledge acquired. These results highlight the need for ITE that is more integrated and links mathematical knowledge with PCK.

CONCLUSION

This research analyzed future primary school teachers' perceptions in Angola, focusing on Numbers and operations with a PCK focus. The analysis revealed variability in how prospective teachers perceive pedagogical strategies. Although they are more familiar with 'Calculation with integers and decimal numbers,' difficulties persist, especially in practical applications, as evidenced by lower performance in 'problem-solving.' These difficulties highlight the need to develop teaching strategies that are aligned with the development of conceptual and operational knowledge of Numbers and operations (Lo, 2020). Consequently, we conclude that there are statistically significant differences between institutions regarding the dimensions analyzed in this work's questionnaire, where pedagogical strategies vary from one institution to another. In addition, the results show a positive correlation between 'Problem-solving' and 'understanding number sense' and a positive but weak correlation between 'problem-solving' and 'calculation with integers and decimal numbers.' These associations may reinforce the need to employ more effective teaching strategies in numbers and operations to facilitate a deeper understanding of this topic.

The study contributes to future teachers' understanding of PCK in teaching numbers and operations in Angola. It highlights how pedagogical strategies are observed and the difficulties associated with their practical application. On the other hand, it shows the importance of teacher training programs being well adapted to the context and needs of the Angolan reality. However, future studies could focus on developing strategies to improve problem-solving methodologies in the first years of ITE education programs. It will also be important to investigate how institutional differences influence the perception and application of pedagogical strategies and explore ways to integrate conceptual, operational, and applied knowledge in teaching Numbers and operations. Additionally, it is recommended that intervention programs be designed to address specific gaps, particularly in problem-solving and conceptual areas, and to evaluate their impact over time. Similarly, qualitative studies could be developed to provide a detailed insight into the experiences and perceptions of future teachers, thereby improving their training in Numbers and operations.

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AUTHORS' CONTRIBUTION

Conceptualization, A.J., P.C. and A.P.A.; data curation, A.J. and M.N.; formal analysis, A.J. and M.N.; investigation, A.J., P.C. and A.P.A.; methodology, A.J. and M.N.; writing-original draft, A.J. and A.P.A.; writing-review and editing, A.J., P.C., M.N. and A.P.A.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

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