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PROMOVER O CÁLCULO DE PRIMITIVAS COM RECURSO AO PERCEPTUAL LEARNING: JOGOS DIGITAIS E ORQUESTRAÇÕES INSTRUMENTAIS

PROMOTING ANTIDERIVATIVE CALCULATION THROUGH PERCEPTUAL LEARNING: DIGITAL GAMES AND INSTRUMENTAL ORCHESTRATIONS

PROMOVER EL CÁLCULO DE PRIMITIVAS MEDIANTE EL APRENDIZAJE PERCEPTIVO: JUEGOS DIGITALES Y ORQUESTACIONES INSTRUMENTALES

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RESUMO

Introdução: O cálculo de primitivas é frequentemente identificado como uma dificuldade para os alunos. A Perceptual Learning (PL), aliada ao uso de artefactos digitais como jogos e orquestrações instrumentais adequadas, pode promover uma melhor compreensão da estrutura das expressões e facilitar esta aprendizagem.
Objetivo: Determinar de que forma artefactos digitais baseados nos princípios da PL, integrados em orquestrações instrumentais, contribuem para a fluência e precisão no cálculo de primitivas pelos alunos.
Métodos: Foi seguida uma metodologia de Design Science Research com três iterações. Participaram alunos portugueses do 12.º ano. Desenvolveram-se jogos digitais centrados na identificação de estruturas de primitivas imediatas. A recolha de dados incluiu testes, observações e interações com os jogos, analisados com recurso a métodos qualitativos e quantitativos.
Resultados: Os alunos evidenciaram melhorias na descoberta e fluência no cálculo de primitivas. Verificou-se que a articulação entre jogos, mediação do professor e atividades de compreensão potenciou a aprendizagem. A integração progressiva dos jogos numa única aplicação, com elementos como feedback imediato e vidas, contribuiu para evitar estratégias de tentativa e erro. Apesar do contexto pandémico, observou-se uma tendência para o trabalho colaborativo.
Conclusão: Jogos digitais concebidos segundo a PL, quando integrados em orquestrações adequadas, mostram-se eficazes no ensino de primitivas. A mediação docente é essencial para que os artefactos se tornem verdadeiras ferramentas epistémicas.

Palavras-chave: cálculo de primitivas; aprendizagem percetiva; jogos digitais; orquestrações instrumentais; ensino de matemática

ABSTRACT

Introduction: The calculation of antiderivatives is often identified as a challenge for students. Perceptual Learning (PL), combined with the use of digital artifacts such as games and appropriate instrumental orchestrations, can foster a better understanding of expression structures and facilitate this learning.
Objective: To determine how digital artifacts based on PL principles, integrated into instrumental orchestrations, contribute to students' fluency and accuracy in calculating antiderivatives.
Methods: A Design Science Research methodology was followed across three iterations. Participants were 12^{th}- grade Portuguese students. Digital games focused on identifying structures of basic antiderivatives were developed. Data collection included tests, observations, and interactions with the games, analyzed using qualitative and quantitative methods.
Results: Students showed improvement in both discovery and fluency in computing antiderivatives. The combination of games, teacher mediation, and conceptual understanding activities enhanced learning. The progressive integration of the games into a single application, with features such as immediate feedback and life-based constraints, helped prevent trial-and-error strategies. Despite the pandemic context, a tendency toward collaborative work was observed.
Conclusion: Digital games designed according to PL principles, when embedded in appropriate orchestrations, prove effective in teaching antiderivatives. Teacher mediation is essential for these artifacts to become true epistemic tools.

Keywords: antiderivative calculation, perceptual learning, digital games, instrumental orchestrations, mathematics education

RESUMEN

Introducción: El cálculo de primitivas suele identificarse como una dificultad para los estudiantes. El aprendizaje perceptivo (Perceptual Learning, PL), combinado con el uso de artefactos digitales como juegos y orquestaciones instrumentales adecuadas, puede favorecer una mejor comprensión de la estructura de las expresiones y facilitar este aprendizaje.
Objetivo: Determinar de qué forma artefactos digitales basados en los principios del PL, integrados en orquestaciones instrumentales, contribuyen a la fluidez y precisión en el cálculo de primitivas por parte de los estudiantes.
Métodos: Se siguió una metodología de Design Science Research con tres iteraciones. Participaron estudiantes portugueses del 12.º curso. Se desarrollaron juegos digitales centrados en la identificación de estructuras de primitivas inmediatas. La recogida de datos incluyó pruebas, observaciones e interacciones con los juegos, analizadas mediante métodos cualitativos y cuantitativos.
Resultados: Los estudiantes mostraron mejoras en el descubrimiento y fluidez en el cálculo de primitivas. Se comprobó que la articulación entre juegos, la mediación del profesor y las actividades de comprensión potenciaron el aprendizaje. La integración progresiva de los juegos en una única aplicación, con elementos como retroalimentación inmediata y vidas, ayudó a evitar estrategias de prueba y error. A pesar del contexto pandémico, se observó una tendencia al trabajo colaborativo.
Conclusión: Los juegos digitales diseñados según los principios del PL, cuando se integran en orquestaciones adecuadas, resultan eficaces para la enseñanza de primitivas. La mediación docente es esencial para que estos artefactos se conviertan en verdaderas herramientas epistémicas.

Palabras clave: cálculo de primitivas, aprendizaje perceptivo, juegos digitales, orquestaciones instrumentales, enseñanza de las matemáticas

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INTRODUCTION

Students' difficulties with the topic of integral calculus are well documented in the literature (Almeida et al., 2021; Domondon et al., 2022; Mahathir et al., 2024). This is a long-standing issue and one that remains hard to resolve. One of the problems identified by Li et al. (2017) is that students struggle to distinguish the patterns of several similar reference functions and, therefore, are unable to apply them to produce the appropriate results. In the context of antiderivatives, it is crucial to recognize their structure—what Li et al. (2017) define as "patterns of several similar reference functions" (p. 26). At Monroe Community College (2024), it is stated: "When approaching a function to integrate, the procedure may not be obvious. However, it is mostly a visual task—that is, the integrand shows what to do: it's a matter of recognizing the shape of the function."

Research shows that visual cognition processing in expert chess players is more efficient due to stored pattern blocks and matches (Küchelmann et al., 2022). An expert in antiderivatives, when looking at an expression, grasps its structure and relates the various sub-expressions it comprises, aiming to fit that function into one of the known integration rules by identifying functions and their derivatives. The core idea of our study is that students can also perceive this structure—just as a chess player does when playing. To this end, we adopt a theoretical framework based on Perceptual Learning (PL), which improves the extraction of information through practice and fosters the perception of structured concepts. Defined by Gibson in 1969, PL refers to long-term improvements in perception as a result of practice or experience (Kellman & Massey, 2013).

In this study, we aim to design and develop digital artefacts (educational games) and instrumental orchestrations to help overcome the identified difficulties in learning antiderivatives (e.g., Almeida et al., 2021; Li et al., 2017). Aligned with these goals, the following research questions (RQs) were formulated:

RQ1 – What features of the artefacts and their instrumental orchestrations support the perception of antiderivative structure and contribute to students' learning?

RQ2 – How did the artefacts, their instrumental orchestrations, teacher mediation, and student collaboration adapt throughout the iterative cycles?

RQ3 – What are the effects of using these artefacts and their instrumental orchestrations on students' learning of antiderivatives?

1. THEORETICAL FRAMEWORK

Visual cognition plays a central role in perceiving structures and patterns, allowing the construction of complex visual entities from partial visual stimuli, based on prior knowledge and inference (Cavanagh, 2011). This ability is evident in experts, such as chess players, who rapidly recognize meaningful configurations, and it can extend to domains such as antiderivative calculation, where perceiving the structure of expressions facilitates their resolution (Küchelmann et al., 2022). This phenomenon aligns with the concept of Perceptual Learning (PL), which involves an active process of information extraction and organization (Kellman & Massey, 2013).

Perception is an interpretive process in which the brain organizes sensory stimuli based on previous experiences, adjusting how information is extracted through practice and improving performance (Frangou et al., 2019). Kellman et al. (2010) identify two core effects in this process: discovery, referring to the identification of relevant information, and fluency, related to the more efficient extraction of that information. These principles underpin the development of Perceptual Learning Modules (PLMs), pedagogical strategies aimed at promoting structural learning through trained perception.

Several studies have explored adaptive strategies to optimize learning based on PL principles. Rau and Wu (2018) demonstrated that activities focused on perception and conceptual understanding have synergistic effects on learning: perception helps to make sense of information, and understanding reinforces perceptual processes. Their work suggests that instructional strategies should combine both approaches.

1.1 Artefacts and Instrumental Orchestrations

The mere presence of digital technologies in mathematics classrooms does not ensure meaningful learning. It requires a process of appropriation known as instrumental genesis (Drijvers et al., 2020), through which the teacher adapts and adopts digital tools according to their instructional intentions. In educational contexts, the distinction between artefact and tool is particularly relevant. An artefact is a material object with a potential purpose; it becomes a tool when it is mobilized by an agent to fulfil that purpose (Lopes & Costa, 2019; 2021). This transformation only makes sense in relation to the user and the task.

Moreover, artefacts do not need to be physical—algorithms or digital games are also artefacts that can become epistemic tools when they promote active student learning (Lopes & Costa, 2021). The appropriation of artefacts by students is mediated by instrumental orchestrations—arrangements planned and adapted by the teacher, which involve the configuration of the learning environment, the modes of artefact exploration, and didactical performance (Drijvers et al., 2010). The orchestra metaphor, often used to describe this process, emphasizes the collaborative nature of teaching: the teacher, like a conductor, guides the students who, even without being experts, actively contribute to knowledge construction (Tabach, 2011).

Several researchers have identified types of instrumental orchestrations, some of which have been extended to distance learning contexts and are synthesized in (Authors, 2020). Studies such as Lopes and Costa (2019) highlight frequent limitations in the educational use of digital artefacts, ranging from the quality of the resources to the lack of robust theoretical foundations for their pedagogical implementation. Furthermore, the teacher's mediation plays a crucial role, defined as the set of actions and languages used to respond to students' learning challenges (Lopes et al., 2010). This mediation involves, among other dimensions, epistemic awareness—how information is structured and the teacher's ability to make real-time decisions—key for responsive teaching that

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adjusts to students' needs, enables immediate pedagogical reformulations, and fosters relevant epistemic practices (Araújo et al., 2019).

1.2 The Use of Games in Education

In mathematics education, games have shown particular effectiveness in promoting student engagement and understanding of abstract concepts (Tokac et al., 2019). They allow for contextualisation of concepts, provide immediate feedback, encourage self-regulation, and foster collaboration (Hui & Mahmud, 2023). Studies show that educational games can significantly improve performance in mathematics, especially in areas such as calculus, logical reasoning, and problem-solving (Khalid et al., 2025). However, the educational impact of games depends on effective orchestration that aligns game elements with learning objectives. When the game's design does not adequately integrate mathematical concepts, there is a risk that the focus shifts to gameplay mechanics at the expense of disciplinary content (Moyer-Packenham et al., 2019).

Additionally, using games in the classroom presents challenges such as teacher resistance, the need for specific training, technological limitations, and difficulties in curricular integration (Hui & Mahmud, 2023). For games to become epistemic tools, they must be embedded in a pedagogical strategy that involves teacher support, timely feedback, and a clear articulation between game mechanics and content (Erhel & Jamet, 2013). Well-designed games can significantly contribute to mathematics learning in both face-to-face and digital contexts, provided they are integrated intentionally, critically, and with teacher mediation (Chen et al., 2022). The studies we found on the development of games for teaching integral calculus were Pramuditya et al. (2019) and Eyrikh et al. (2020). The first uses a game with characters and various levels, in which students need to give answers involving definite and indefinite integrals; the second one focuses on the computation of the area under a curve (integral calculus). In 2025, Authors published a first study, where we developed artefacts (games) and instrumental orchestrations to help students develop the ability to compute primitives, aligned with the principles of PL. In this study, we bring new features to the games, refined instrumental orchestrations and a new implementation with another class of students. We also bring the data corresponding to this new implementation.

2. METHODS

2.1 Research Design

This paper presents part of a more comprehensive study which methodological approach used was Design Science, operationalized through Design Science Research Methods (DSRM) (Peffers et al., 2008) and Case Study methodology (Cohen et al., 2018). The study was conducted with two 12th-grade mathematics classes and their mathematics teacher from a Portuguese public school during the academic years 2019/2020 and 2020/2021. The implementation of this methodological approach (Figure 1) involved a multi-step process with several iterations.

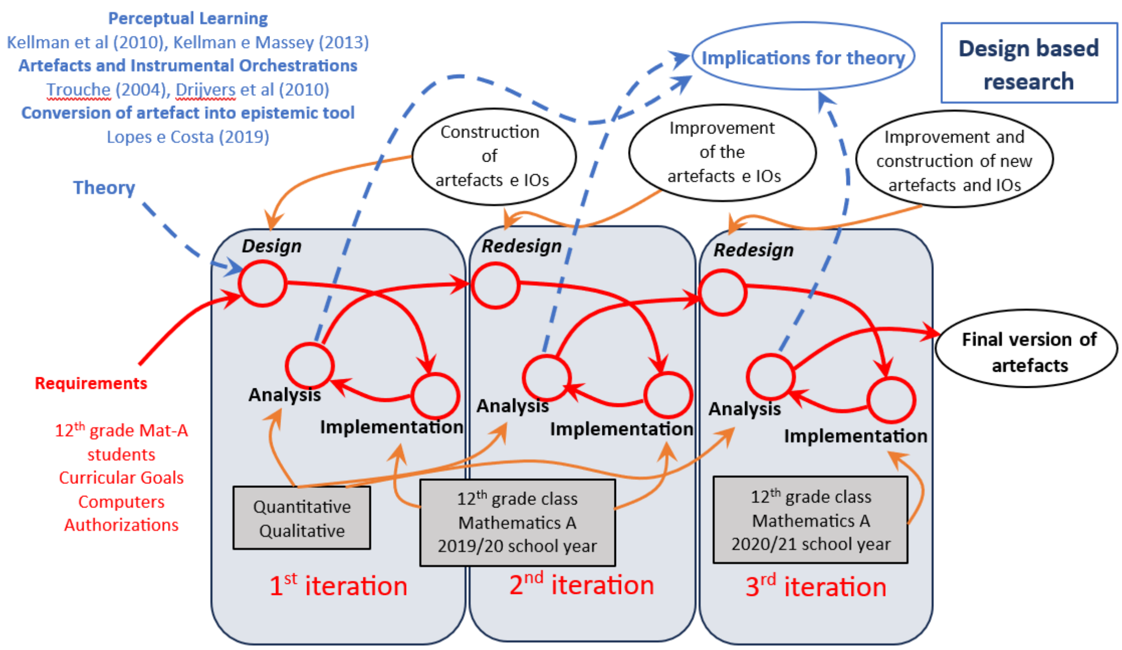

Figure 1 – Design research

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The starting point for the development of the artefacts (games) was the construction of a document explicitly detailing the reasoning process behind calculating antiderivatives, validated by a panel of experts (mathematicians and mathematics educators). Based on the theoretical framework of PL (Kellman & Massey, 2013), an iterative process was launched, including the design and development of the artefacts (games) (Table 1) and their respective instrumental orchestrations.

Table 1 - Articulation between Design Based Research and Case Study

Iteration 1 13/05 a 26/05 de 2020 Case Study 1		Iteration 2 24/06 de 2020 Case Study 2		Iteration 3 20/04 a 30/04 de 2021 Case Study 3	
Year/class	Artefacts	Year/class	Artefacts	Year/class	Artefacts
12th grade / X	Memory Game Build Your Antiderivative I – V1 Build Your Antiderivative II – V1	12th grade / X	Build Your Antiderivative I – V2 Build Your Antiderivative II – V2	12th grade / Y	Memory Game Build Your Antiderivative – V3 Game 1 Game 2 Game 3

Caption: Vi means version i

Demonstration during the three iterations was carried out through case studies. The evaluation in the first iteration was conducted after data collection and analysis, which led to modifications in the artefacts. A second iteration was then conducted within the same academic year, one month after the first. At the end of the school year, data were collected and analysed, leading to the development of a new type of instrumental orchestration (Authors, 2020), the artefacts Memory Game and Build Your Antiderivative 1 – V1 and Build Your Antiderivative 2 – V1 (Authors, 2021), and the instrumental orchestrations used for teaching antiderivatives with the mentioned artefacts. Their impact on students' ability to calculate antiderivatives was also analysed (Authors, 2026). The analysis indicated that further improvements to the artefacts were needed, including the creation of a new game. In this paper we present a third iteration (Figure 1) which was conducted with another class, in the 2020/2021 school year, from which data were collected and analysed, allowing the refinement of the artefacts and instrumental orchestrations.

2.2 Data collection instruments

Data were collected through audio and video recordings of both in-person and remote classes. Students' interactions with the games were captured by recording their computer screens. To assess learning progression, pre- and post-tests composed of 12 questions on antiderivative calculation were administered during the 1st and 3rd iterations. In the 3rd iteration, these tests were repeated at five different moments (AV_1 to AV_5) to allow for a more precise analysis of learning progress. At the end of the 1st and 3rd iterations, a summative assessment on the topic of antiderivatives and integral calculus was applied.

To gather students' opinions and suggestions regarding the games, online questionnaires were used. These questionnaires included both open-ended and closed-ended questions, allowing for the evaluation of overall opinion, perceived level of difficulty (using a Likert scale), relevance of the game (with justification), usefulness in learning, what was learned, and whether students would play the game again. Additionally, students were invited to suggest improvements to the games.

2.3 Data analysis

Qualitative data (questionnaires, lesson plans, and classroom recordings) were analysed using content analysis (Bardin, 2021). Based on the lesson planning and classroom practice data, we examined the didactic performance and the modes of instrumental orchestration. The analysis focused on the roles of both teacher and students. Regarding the teacher, we emphasised mediation, particularly in terms of how information was presented and processed, and the teacher's awareness in making real-time decisions in the classroom. Concerning the students, we analysed how they used the games and the extent of collaboration among them.

For quantitative analysis, statistical tests were conducted on the five successive assessments to determine whether students' ability to calculate antiderivatives improved. All differences between the post-test and each of the previous tests were calculated, resulting in paired difference variables for the five successive assessments. Normality was tested using the Shapiro–Wilk test, showing that the differences followed a normal distribution except for the difference between AV_5 and AV_2. Therefore, paired-samples t-tests were performed to assess whether the mean differences were significantly different from zero, and the Wilcoxon non-parametric test was used for AV_5–AV_2. All analyses were conducted using IBM SPSS Statistics for Windows, Version 22, considering differences statistically significant at $p \leq 0.05$.

Screen recordings from the Memory game were used to count the number of attempts (flips) and the time each student took to complete the game. In Build your Antiderivative games 1.1 and 1.2, we checked whether students completed the game, recorded the time, and assessed performance quality. Game quality was inferred from a maximum score of 750 points by subtracting the score obtained and dividing the result by 5 to estimate the number of errors. In games 2 and 3 of Build your Antiderivative, time, score, and number of attempts were recorded, and game quality was inferred using a similar method.

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3. RESULTS

3.1 Memory Game and Build Your Antiderivative Game Artefacts and Instrumental Orchestration

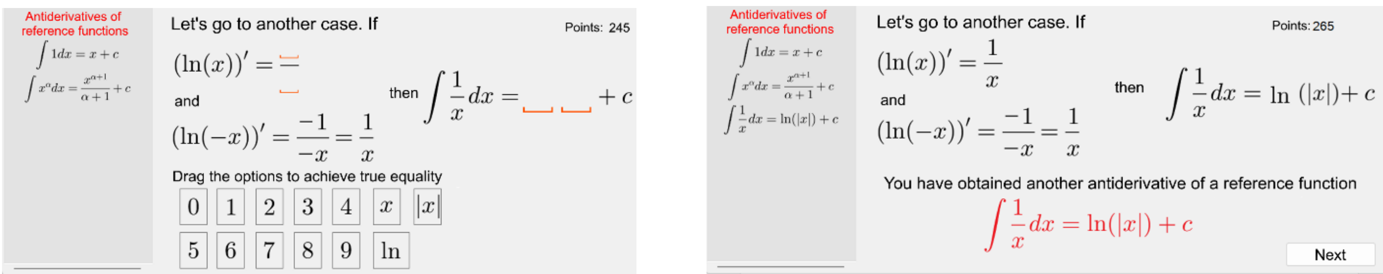
The Memory Game[1] is an online card-matching game involving function expressions, where the objective is to form pairs of cards such that one expression is the derivative of the other (and, consequently, one is the antiderivative of the other, up to a constant). The Memory Game was implemented as described in (Authors, 2021), yielding results analogous to those reported in (Authors, 2026). Students' opinions collected via questionnaires were consistent with those from the first iteration, and thus no changes were made. Regarding the questionnaires on the level of difficulty, most students considered the Memory Game balanced, with 54% classifying it as “neither easy nor difficult”, while 31% considered it “difficult” and 15% “easy”.

The Build Your Antiderivative game[2] (Figure 2) emerged as a result of two iterations of the previous Build Your Antiderivative 1 and Build Your Antiderivative 2 games, as described in (Authors, 2021).

Iteration 1 and 2		**Build Your Antiderivative I** ↓	**Build Your Antiderivative II** ↓
Iteration 3	**Game 1**	**Game 2**	**Game 3**
	• Reasoning to obtain: ○ antiderivatives of reference functions ○ rules of antiderivation (basic antiderivation) • Get a form	• Visualize the different forms/structures that the basic antiderivative can have • Obtain $\int u'(x)f(u(x))dx$ • 1 option out of 3	• Obtain $\int u'(x)f(u(x))dx = F(u(x)) + c$ • 3 options out of 6

Figure 2 – Build Your Antiderivative game features

As a result of the iterative process, it was concluded that the Build Your Antiderivative game could be improved to enhance its potential as an epistemic tool for students. Thus, the new Game 1 aims for students to discover the antiderivatives of reference functions (Game 1.1) and the antiderivation rules (Game 1.2) through the explicit reasoning required to construct these expressions. This is achieved initially by having students build each antiderivative of the reference functions, dragging each letter or digit that composes these expressions to the correct place, according to differentiation rules (Figure 3). Through this process, students progressively construct each expression on the left-hand side of the screen, thereby assembling a reference sheet. Finally, they apply this to various expressions involving linear combinations of reference functions.



[1] The Memory game levels 1 and 2 are available at https://matchthememory.com/derivadas-nivel1 and https://matchthememory.com/experiencia, respectively.

[2] The Build Your Antiderivative game installation is available at https://drive.google.com/drive/folders/1zRP6scC9dGL1-IopFD0TCnMS0PRrlXRU?usp=sharing.

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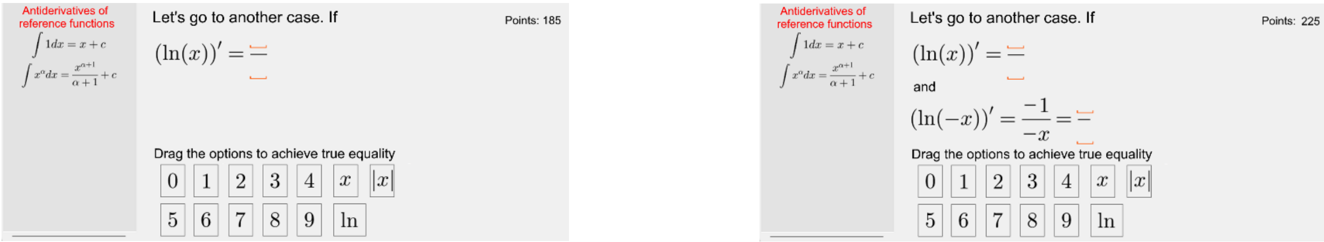

Figure 3 - Game 1.1 example – explanation of the antiderivative of the reference function $\int \frac{1}{x} dx$ and $\int \frac{1}{-x} dx$. The various equalities of the concrete example

Game 1.2 (Figure 4) has the same purpose and operates in the same way, but focuses on the antiderivation rules involving composite functions, commonly referred to as basic or immediate antiderivatives.

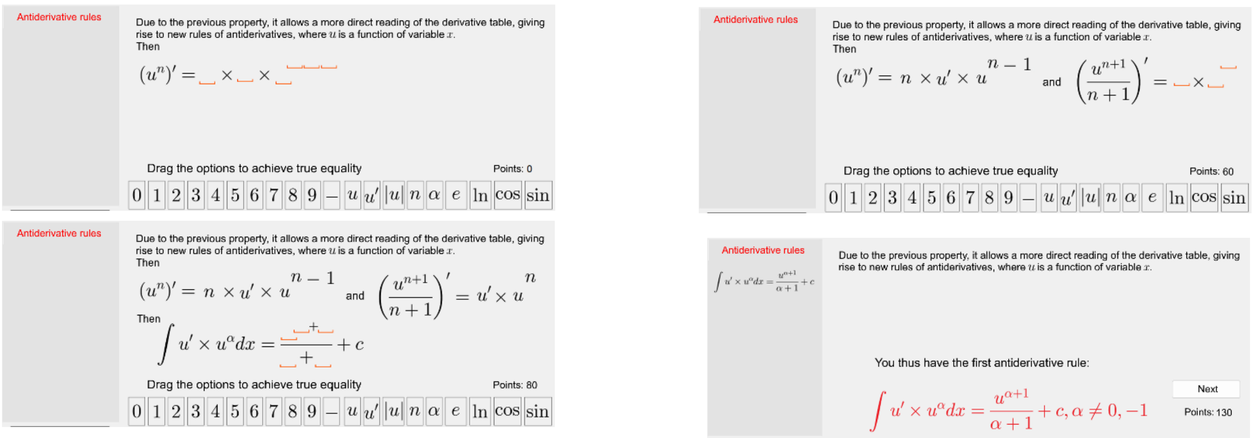

Figure 4 – Game 1.2 example - explanation of a tiderivation rule (in this case, $\int u' \times u^n \, dx$)

Game 2 of Build Your Antiderivative aims to develop students' ability to visualize the different forms an immediate antiderivative can take, as proposed in Perceptual Learning. To achieve this, the student must analyse the structure of the given expression and select, among the available options, the one that transforms it into an immediate antiderivative. The goal is for the student to develop greater sensitivity to the structure of antiderivatives, correctly identifying their components and establishing relationships between them. Faced with an incomplete expression of an immediate antiderivative, the student must complete it by choosing one of three multiple-choice options. More details in (Authors, 2021). As a result of the iterative process, the game levels were divided into two levels of difficulty.

Game 3 presents a greater challenge. The student is shown an equation in which the left-hand side contains an antiderivative of a product or quotient, and the right-hand side presents the result. The aim is to select three expressions, from six available options, to form a true equality. More details in (Authors, 2021). However, as a result of the iterative process, five lives were introduced in this game, since students were previously solving it through trial and error.

Based on screen recordings collected during Game 1 of Build Your Antiderivative, all students completed both Game 1.1 and Game 1.2. On average, students took 9 minutes and 44 seconds to complete Game 1.1 and scored 722 out of a possible 750 points, making fewer than 6 mistakes on average, equivalent to an 8% error rate.

In Game 1.2, all students again completed the game. On average, they scored 311 out of 350 points, suggesting fewer than 4 mistakes, or an 11% error rate. The average time to completion was 7 minutes and 10 seconds. Overall, student feedback was very positive. They described the game as interesting, interactive, motivating, and useful for learning antiderivatives, highlighting its dynamic and competitive nature. Most students felt the game offered an easier and more enjoyable way to learn, fostering engagement with the content. Regarding the difficulty level, half of the students rated the game as moderately difficult, 25% as easy, and the remaining 25% as difficult. All participants considered the game relevant for learning antiderivatives, stressing its practical connection to the content and its motivational nature.

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The learning sequence presented in the game was generally very well received. Only one student expressed some reservations, while others highlighted the gradual progression in difficulty and the initial review of derivatives as facilitators of understanding.
All students found the game useful for learning antiderivatives. Only one student stated they would not play it again, while the rest expressed willingness to use it again, particularly when needing to review the topic or deepen their understanding.
Suggestions for improvement included three main points: improving the layout, optimizing game performance with more engaging mechanics, and including more levels for each antiderivative rule.
As for Game 2, results and student feedback were consistent with those reported in (Authors, 2021). Concerning the difficulty level for Game 2, 40% of the students classified it as “neither easy nor difficult”, 50% as “difficult”, and 10% as “very difficult”.
Regarding Game 3, the introduction of lives discouraged trial-and-error strategies. Students needed, on average, two attempts to complete the game, taking 20 minutes and 37 seconds on average. In their final (successful) attempt, students scored an average of 476 out of 540 possible points. Based on scores as a proxy for “game quality,” students made an average of 29.3 mistakes across all attempts. Regarding the difficulty level for Game 3, 37% of the students considered it “neither easy nor difficult”, while 63% rated it as “difficult”.
Regarding the instrumental orchestrations, the combination of direct instruction and perceptual learning through games proved effective in this study. The PowerPoint presentation, which supports the teaching of this topic, completes the foundational structure of this orchestration. Figure 5 schematically presents the refined exploration mode resulting from the DSRM process.

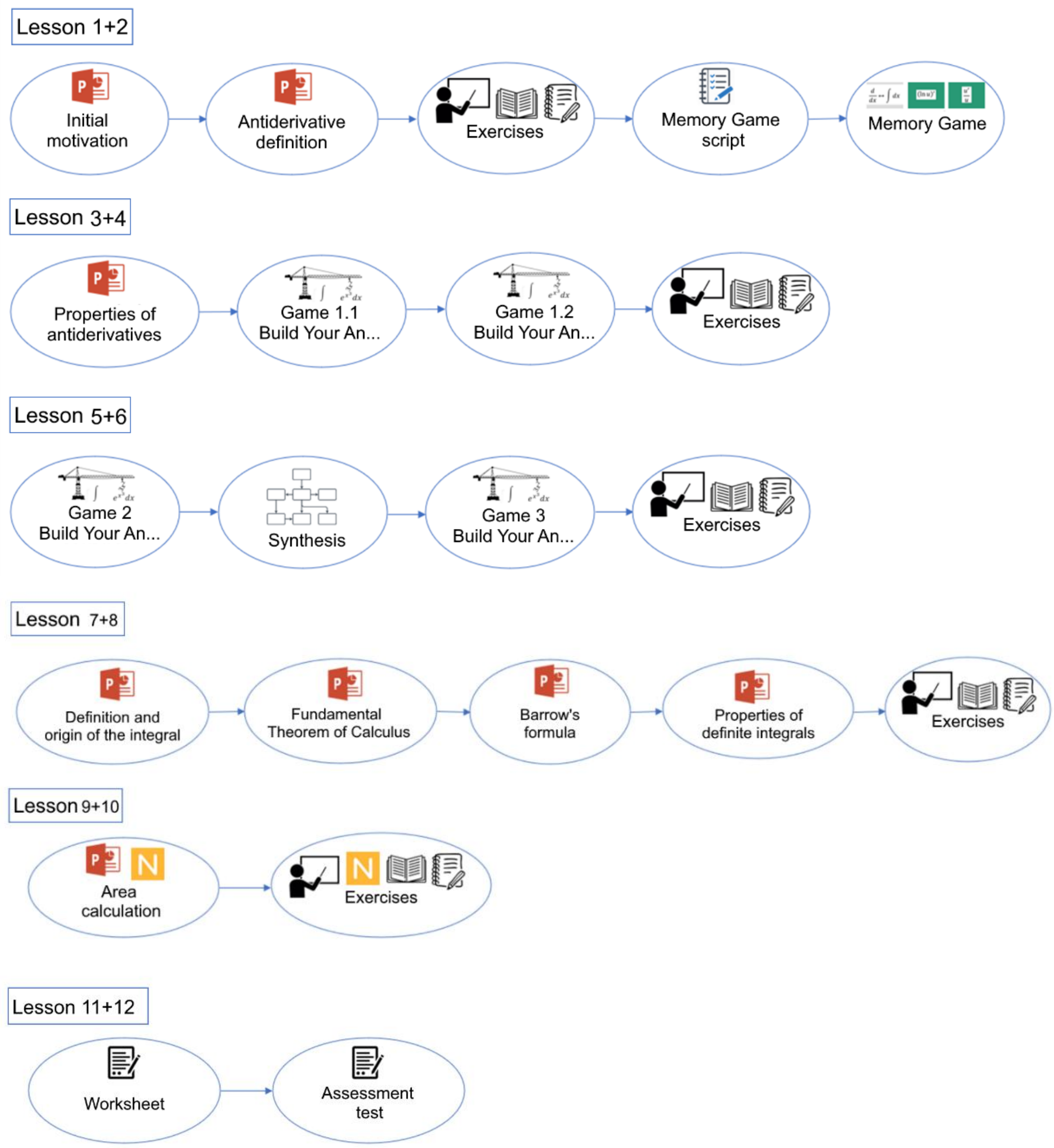

Figure 5 – Recommended exploration mode for approaching the chapter on primitives and integral calculus

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3.2 Implications for Teaching Performance

The analysis of lesson planning and classroom practice revealed that the teacher's mediation was essential. The teacher provided explanations and reacted promptly upon realizing that most students were struggling, adjusting the lesson plan accordingly. This demonstrates attentiveness and the ability to adapt in real time.

Regarding the use of the games, some students demonstrated autonomy—for example, completing the game independently or even creating a solution guide. However, several students had difficulty applying mathematical concepts, which limited the game's effectiveness for them. There was collaboration among some students, who gathered to exchange ideas or observe each other's progress. Although spontaneous and unplanned, this interaction helped resolve doubts in some cases.

The initial plan assumed that the sequence of Games 1.1 and 1.2 would be sufficient for students to internalize the concept and technique of antiderivation. However, this was not confirmed in practice, as students encountered difficulties completing Game 2.

3.3 Implications for Learning

The results obtained from the five successive assessments allow us to conclude that all the paired differences under analysis can be considered non-zero (max(Sig.) = 0.034 < 0.05). Since the test statistics are positive, this indicates that the results of each subsequent test were consistently higher than those of the previous tests, including for diff AV2–AV5. From this statistical analysis, we conclude that each assessment result was significantly and progressively better than the one before. Next, we tested whether the differences between tests could be assumed to exceed zero and specifically examined whether increases of 10 or 20 points could be validated.

It was found that an average increase of more than 20 points between two consecutive assessments was statistically significant (Sig. = 0) — except between the last two (AV4 and AV5 (Sig. = 0.82), where the increase was smaller.

Overall, in terms of difficulty, the students perceived a progressive increase in difficulty across the games, suggesting that the sequence provided increasingly challenging tasks while remaining pedagogically appropriate.

Concerning the final test, scored from 0 to 200, the average score was 180, the median was 185, the minimum score was 145, and the maximum was 200.

4. DISCUSSION

We begin by summarising the key findings from the three cycles. In the first iteration, it was shown that the instrumental orchestrations and artefacts (games) were useful for developing students' ability to calculate antiderivatives. However, some gaps were identified and addressed in the second and third iterations. In the third iteration, it was expected that playing Game 1 would prepare students to successfully complete Game 2, but this did not occur. The initial difficulty in completing Game 2 revealed that students had not yet fully grasped the process of finding antiderivatives. The teacher's intervention—by adapting the planned lesson and incorporating a more traditional problem-solving approach—together with teacher mediation and prior perceptual learning experiences, played a decisive role in students' progress. These results suggest that combining conceptual understanding activities with PL-based tasks enhanced learning outcomes, as advocated by Rau and Wu (2018). Moreover, despite the pandemic context, students attempted to collaborate informally, against the teacher's instructions. This suggests that in a non-pandemic context, it would be relevant to test a didactic configuration that explicitly includes collaborative student work.

We now discuss the theoretical and practical contributions of this study that are relevant to improving students' skills in calculating antiderivatives.

Contribution 1: Artefacts and their instrumental orchestration as facilitators of students' perception of antiderivative structure

(i) the intentional construction of various artefacts, especially games, designed to support antiderivative learning based on PL principles;

(ii) an approach to teaching and learning antiderivatives (instrumental orchestration and digital artefacts – games), combining direct instruction (focused on conceptual understanding) with digital games (focused on perceptual learning).

The two new PLMs developed — Memory Game and Build Your Antiderivative — aimed to reinforce students' fluency in computing antiderivatives, in line with PL (Kellman et al., 2010). These PLMs offer students multiple tasks at different levels with immediate feedback, allowing them to extract relevant information selectively, reduce cognitive load, and identify higher-order invariances in expression patterns (Kellman et al., 2008). The observed effects include improved pattern recognition (discovery) and greater speed (fluency), as proposed by Kellman et al. (2010). The developed artefacts and instrumental orchestration may also be useful for first-year university teachers, given the persistent difficulties in learning antiderivatives (Almeida et al., 2021; Domondon et al., 2022) and the importance of mastering them for future learning (Almeida et al., 2021).

Contribution 2: Teacher mediation, artefact use, and collaborative student work as essential elements

The aim was for the digital games to act as epistemic tools. For this purpose, a new game (Game 1) was developed in which students construct antiderivative rules and apply them to linear combinations of reference functions. However, the initial implementation of this game proved insufficient without teacher mediation. The teacher's intervention—interrupting the planned

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sequence and turning to traditional exercise-solving—was crucial to student understanding, confirming that digital artefacts alone do not guarantee educational benefits. Nevertheless, Game 1 partially fulfilled its epistemic purpose, as students did not need to relearn the rules, only their application. Additionally, even during the pandemic, students sought ways to collaborate, highlighting the importance of including explicit collaborative work in future iterations.

Contribution 3: Combining conceptual understanding and PL to work on antiderivative structure proved effective for student learning

The implementation of PL-based digital games was effective in promoting fluency and structural understanding in computing immediate antiderivatives. The significant improvement in time and number of attempts between the first and second iterations of Build Your Antiderivative I suggests long-lasting learning gains in fluency, consistent with the findings of Kellman et al. (2008).

The progressive structure of the game, with levels organised by increasing difficulty, followed PL principles and enabled students to identify regularities and distinguish relevant from irrelevant variations, promoting a deeper understanding of antiderivative rules. The visual artefacts, offering immediate feedback and minimal instruction, proved to be effective epistemic tools, helping students perceive underlying structures and apply rules correctly.

This contribution shows that combining traditional teaching, conceptual understanding, and PL principles—as suggested by Rau and Wu (2018) and Kellman et al. (2008)—enhanced the development of durable skills in computing immediate antiderivatives.

CONCLUSION

This study showed that the digital artefacts Memory Game and Build Your Antiderivative, created based on Perceptual Learning (PL) principles, when integrated with conceptual understanding activities and supported by a tailored instrumental orchestration, promote meaningful learning in the calculation of antiderivatives. By facilitating the recognition of mathematical structures and patterns, these games help overcome difficulties associated with symbolic abstraction and foster a more visual and interactive learning experience.

The final instrumental orchestration proved essential for effectively integrating the artefacts. The teacher's mediation — supporting the implementation of the games, identifying difficulties and proposing alternative approaches to overcome them, keeping students focused on the task, and demonstrating flexibility in accepting collaborative work among students — played a decisive role in the learning success.

This study reinforces the importance of classroom practices that combine educational technology, guided exploration, and an emphasis on mathematical structure, particularly in traditionally challenging domains such as the teaching and learning of antiderivatives.

AUTHORS' CONTRIBUTION

Conceptualization, C.M.; data curation, C.M.; formal analysis, C.M. and C.C.; investigation, C.M.; methodology, C.M. and C.C.; validation, C.C.; visualization, C.C.; writing – original draft, C.M. and C.C.; writing – review & editing, C.M. and C.C.

CONFLICT OF INTEREST

The authors declare no conflict of interests.

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