THE INFLUENCE OF THE SAMPLING METHOD ON QUALITY CONTROL IN THE PRODUCTION OF AMMUNITION

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ABSTRACT

In statistical process control, particularly in the production of ammunition, it is assumed that the quality characteristic under study follows a normal distribution, with the sampling method used being one of the critical factors. Regarding sampling methods, among fixed, dynamic and mixed methods, some are more effective than others for different magnitudes of quality change. In this study, a dynamic method based on the Laplace function was used to define the sampling intervals, called LSI (Laplace Sampling Intervals). Using analytical expressions of the statistical properties of the sampling method, we consider that the quality characteristic follows a gamma distribution (asymmetric) and Shewhart-type control charts for sample averages. In this context, the performance of the LSI sampling method, in terms of system failure times (AATS, Adjusted Average Time to Signal) is compared with the performance of the FSI (Fixed Sampling Intervals) and VSI (Variable Sampling Intervals) sampling methods, for different values of the shape and scale parameters of the Gamma distribution. The results show the method is robust in detecting quality changes in critical situations and contexts.

Keywords: Statistical Process Control, Adaptive Sampling, X-bar Charts, Gamma Distribution, Production of Ammunition

RESUMO

No controlo estatístico do processo, e em particular na produção de munições, considera-se que a característica da qualidade em estudo tem distribuição normal, sendo o método de amostragem utilizado um dos fatores críticos. Relativamente aos métodos de amostragem, entre métodos fixos, métodos dinâmicos e métodos mistos, uns são mais eficazes que outros nas diferentes magnitudes de alteração da qualidade. Neste estudo utilizamos um método dinâmico que tem por base a função de Laplace para definir os intervalos de amostragem, denominado LSI (Laplace Sampling Intervals). Usando expressões analíticas das propriedades estatísticas do método de amostragem, consideramos que a característica da qualidade segue uma distribuição Gama (assimétrica) e cartas de controlo para médias amostrais, tipo Shewhart. Nesse contexto, o desempenho do método de amostragem LSI, em termos dos tempos de mau funcionamento de um sistema (AATS, Adjusted Average Time to Signal), será comparado com os desempenhos dos métodos de amostragem FSI (Fixed Sampling Intervals) e VSI (Variable Sampling Intervals), para diferentes valores dos parâmetros de forma e de escala da distribuição Gama. Os resultados obtidos mostram que o método é robusto a detetar alterações da qualidade em situações e contextos críticos.

Palavras-chave: Controlo Estatístico do Processo, Amostragem Adaptativa, Cartas de médias, Distribuição Gama

1. INTRODUCTION

In 1924, Shewhart presented a study with the first average control chart, which became popular for assessing the variability in the quality of products and services. This type of chart is widely used in Statistical Process Control (SPC) because it can distinguish between the variability inherent in the process and the variability influenced by external factors. Therefore, the type of chart to be used, which depends on the operational context, plays a key role in the design of a quality control system. This design also includes determining the sampling times, the sample size and the multiples of the standard deviation to be used in the control limits.

In classic Shewhart charts, sampling times, sample sizes, and control limits are fixed throughout the process, referred to as FSI (Fixed Sampling Intervals). However, the FSI sampling method is not very effective in detecting small and moderate changes in quality. To overcome this lower efficiency, other sampling statistics are often used (such as truncated averages, medians, or total medians), and new sampling methods have been developed over the years. Some methods, where the sampling times are fixed at the beginning of the process, are known as predefined sampling (e.g. Banerjee & Rahim, 1988; Rahim & Banerjee, 1993; Lin & Chou, 2005; and Rodrigues Dias & Infante, 2008). Others that update the parameters throughout the process are known as adaptive sampling (e.g. the VSI sampling method – Variable Sampling Intervals (Reynolds et al., 1988), the VSS – Variable Sampling Size (Costa, 1994), the VSSI – Variable Sample Size and Sampling Intervals (Prabhu et al., 1994), the VP – Variable Parameters (Costa, 1999b), and the NSI sampling method – Normal Sampling Intervals (Rodrigues Dias, 1999a).

Initially, control charts are used for averages, but later, sensitivity and robustness studies are performed on the methods using different control charts (EWMA, CUSUM, p-charts, c-charts and np-charts) and different scenarios for the quality characteristic(s) under study (see, for example, the studies by Wan et al., 2023, and Jiang et al., 2024.

In general, sampling methods are developed on the assumption that the quality characteristic under study has a normal distribution, which is not always the case. It is therefore useful to evaluate the performance of sampling methods in different contexts, i.e., when the characteristic under study is not symmetrical and deviates from the normal distribution. Thus, one method can be said to be more robust than another if it shows better statistical performance, usually in terms of its effectiveness in detecting changes in quality. This topic has received the attention of researchers, including the work of Amin & Miller, 1993; Lin & Chou, 2007; Figueiredo & Gomes, 2009; Schoonhoven & Does, 2010; Lin & Chou, 2011; Ou et al., 2012, and Mishra et al., 2019, among others, who study the robustness of different sampling schemes considering that the quality characteristic is not modelled by a normal distribution and sometimes considering different sampling statistics.

In most studies, the statistical performance of the methods is compared in terms of the speed with which significant causes are detected. In this paper, we use the average time of system malfunction, known in the literature as AATS (Adjusted Average Time to Signal) due to the use of adaptive sampling.

In the following sections, we begin with a brief presentation of the LSI method and its main statistical properties. We then describe the FSI and VSI methods and compare the performance of the three methods using AATS. The next point is a study of the robustness of the LSI and VSI sampling methods in the context of non-normality of the quality characteristic, considering that it has a Gamma distribution with different shape and scale parameters. In the cases studied, the effectiveness of the different methods is compared by means of AATS, using simulation when necessary, and an example of practical application in the quality control of ammunition production is presented. Finally, conclusions are drawn, some limitations are shown and proposals for future work are made.

2. THE LSI SAMPLING METHOD

Let μ_0 and σ_0 be, respectively, the mean and standard deviation of the quality characteristic X, which we consider having a normal distribution. Let t_i be the sampling time of order i and $\bar{x}_i$ be the mean of the sample analyzed at that time. According to the LSI sampling method (Carmo et al., 2018), the next sampling instant (of order i+1) is given by

$$t_{i+1} = t_i + \frac{k \cdot e^{-|u_i|}}{2}, \tag{1}$$

where $u_i = \frac{\bar{x}_i - \mu_0}{\sigma_0}\sqrt{n}, \quad t_0 = 0, \quad t_1 = \frac{k}{2}, \quad \bar{x} = \mu_0$, $-L < u_i < L$,

n is the sample size, and k is a convenient scaling constant that depends, in particular, on the costs associated with the production process. Considering that u_i is the average of the reduced sample, if $|u_i| > L$, we are in an out-of-control or false alarm situation. Thus, the sampling intervals, $d_i = t_i - t_{i-1} = k.l(u_{i-1})$, $i = 1,2,3,\ldots$, where l(.) is the probability density function of the reduced Laplace distribution, are independent and identically distributed with the same distribution as a generic variable D defined by

$$D = t_{i+1} - t_i = \frac{k \times e^{-|u_i|}}{2}. \tag{2}$$

The idea behind the adaptive and continuous method is to reduce the sampling frequency when the averages are close to the center line and to increase it in the opposite case, i.e., when the quality is more likely to change. It is an adaptive method in which the time interval until the next sample depends on the information collected in the current sample, as in the VSI method. However, VSI traditionally considers two sampling intervals, while LSI has an infinite number of possible values for the sampling intervals. In practical terms, unlike other adaptive methods, we only need to determine a scale constant k (considering fixed control limits) for the method to be defined.

On the other hand, with the scientific and technological training of human resources and today's computer capabilities, it is quite easy to calculate the average of a sample and determine just-in-time the next sampling time. With this method, as sampling times tend to decrease, the operator's perception that something negative is happening to the product quality increases. This feature of the LSI method could be seen as a disadvantage compared to other methods, but with the ease of obtaining sampling instants, the type of process you want to control (for example, a military production process for nanotechnology components or the collection of signals in transmissions), and the desired objectives, it could become a competitive advantage. In addition, the shortest sampling interval obtained with LSI sampling is identical to the shortest sampling interval most commonly used with VSI sampling, which discourages the most pessimistic about the difficulties of practical application.

Given the assumptions for (1) and (2), a control chart for averages, and that, after a change in the process, μ_0 and σ_0 can take on values $\mu_1 = \mu_0 \pm \lambda\sigma_0$ and $\sigma_1 = \rho\sigma_0$, where λ and ρ are respectively the coefficients of change in average and standard deviation, we obtained for the average sampling interval, E(D), the algebraic expression

$$E(D|\lambda,\rho,n) = \frac{k}{2\beta}\left[e^{\lambda\sqrt{n}+\frac{\rho^2}{2}} \times A(L,\lambda,\rho,n) + e^{-\lambda\sqrt{n}+\frac{\rho^2}{2}} \times B(L,\lambda,\rho,n)\right], \tag{3}$$

where

$$A(L,\lambda,\rho,n) = \Phi\left(\frac{-\rho^2-\lambda\sqrt{n}}{\rho}\right) - \Phi\left(\frac{-L-\rho^2-\lambda\sqrt{n}}{\rho}\right),$$

$$B(L,\lambda,\rho,n)=\Phi\left(\frac{L+\rho^2-\lambda\sqrt{n}}{\rho}\right)-\Phi\left(\frac{\rho^2-\lambda\sqrt{n}}{\rho}\right), \tag{4}$$

and where β is the probability of making a type II error and Φ(u) is the distribution function of the reduced normal.

The expression (3) is a function of n, the coefficient of the control limits L, of λ and ρ, and of β but does not depend directly on the values of the mean or the standard deviation of the quality X. If the process is under control, $\lambda = 0$ and $\rho = 1$, the average sampling interval is given by

$$E\left(D\middle|L\right)=\frac{\sqrt{e}\,k}{\beta}\left[\Phi\left(L+1\right)-\Phi\left(1\right)\right]. \tag{5}$$

Thus, equating (5) to the fixed interval (considering, without loss of generality, the unit sampling period in the FSI method), the constant k is given by the expression

$$k=\frac{\beta}{\sqrt{e}\left[\Phi(L+1)-\Phi(1)\right]}, \tag{6}$$

is equal to 3.8134 if we consider $L = 3$, so the method is defined.

Now consider the time interval between the start of the process and the emission of an out-of-control signal (or false alarm). Its average value, ATS (Average Time to Signal), is given by

$$ATS_{LSI}=k\times l(0)+\left(\frac{\beta}{1-\beta}\right)E\left(D\middle|\lambda,\rho\right), \tag{7}$$

with A(L, λ, ρ, n) and B(L, λ, ρ, n) defined in (4), where we assume that the process starts under control and we consider the longest interval for the first sampling instant. On the other hand, if we consider that the process starts out of control, we consider the shortest interval for the first sampling, obtaining an expression similar to (7), replacing l(0) by l(L). In general, processes start under control, with the failure occurring during the production process, i.e., in a time interval between the taking of two samples. In these situations, the time interval between the process failure and its detection by the control chart is of great importance for the effectiveness of the chart, making it necessary to make an adjustment to the ATS value.

Thus, G is considered as the time interval between the moment the process failure occurs and the moment the first sample is analyzed. The average system failure time, known as AATS, is given by

$$AATS_{LSI} = E(G) + \left(\frac{\beta}{1-\beta}\right) \times E(D), \tag{8}$$

where β is the probability of making a type II error, E(D) defined in (3) and E(G) being (Carmo et al., 2018) given by

$$E(G|L) = \frac{ke^{3/2}}{4} \frac{\Phi(L+2) - \Phi(2)}{\Phi(L+1) - \Phi(1)}, \tag{9}$$

Obtained under the conditions considered in Reynolds et al., (1988) for the VSI method.

2.1. THE FSI AND VSI SAMPLING METHODS

Taking samples at fixed times (e.g., every hour, d = 1) is convenient, but the performance of the FSI sampling method is sometimes unsuitable for highly developed and complex production contexts, and it is considered unsatisfactory for detecting small and moderate changes in a production process.

In the work of Rodrigues Dias, (1987), and in the context of perfect system inspections, an approximation is obtained for the mean malfunction time in the FSI sampling method, which is given by

$$AATS_{FSI} \cong \frac{d}{2}. \tag{10}$$

For dynamic methods, Reynolds et al., (1988) proposed the VSI sampling method. Using two sampling intervals ($d_1 < d < d_2$) and dividing the continuation region,]-L, L[, of the control chart into two sub-regions (a central one:]-w, w[and warning one:]-L, -w[∪]w, L[), the method makes it possible to anticipate (using d_1) or delay (using d_2) the taking of the next sample. Although the method allows the use of more than two sampling intervals, justifications are given for the use of two intervals (Runger & Pignatiello, 1991). Considering two sampling intervals, the authors obtained the following expression

$$W = \Phi^{-1}\left[\frac{2\Phi(L)\times(d-d_1)+d_2-d}{2(d_2-d_1)}\right], \tag{11}$$

when the average sampling interval in VSI, under control, is equal to the sampling period in FSI, allowing the sub-regions to be defined.

According to Reynolds et al., (1988), the average system malfunction time, AATS, is given by

$$AATS_{VSI} = \frac{d_1^2 p_{01} + d_2^2 p_{02}}{2(d_1 p_{01} + d_2 p_{02})} + \left(\frac{\beta}{1-\beta}\right)\times E(D)\ , \tag{12}$$

where $p_{01} = 2[\Phi(L) - \Phi(w)]$ and $p_{02} = 2\Phi(w) - 1$ are the probabilities of $\bar{x}_i \in$]-L, L[, when the process is under control and E(D) is the average sampling interval in VSI method.

2.2. ROBUSTNESS OF LSI METHOD WHEN QUALITY HAS A GAMMA DISTRIBUTION

As mentioned above, in practical applications, the quality characteristic under consideration does not always have a normal distribution. In order to assess the effectiveness and robustness of the LSI method, we will consider different levels of deviation from normality, as was done in the work of Stoumbos & Sullivan, 2002; Figueiredo & Gomes, 2004; Lin & Chou, 2007; Schoonhoven & Does, 2010; Lin & Chou, 2011; Panthong & Pongpullponsak, 2016 and Mishra et al., 2019, where different distributions, different types of graphs, and sometimes other statistics such as median and mean absolute deviation are used.

In this work, we use a Shewhart chart of averages and propose three situations that represent other situations that clearly deviate from normality. Considering that the quality characteristic is modeled by a Gamma distribution with parameters a and b, respectively shape and scale parameters, the probability density function is given by

$$f(x) = \frac{x^{a-1} \times e^{-\frac{x}{b}}}{b^{a} \times \Gamma(a)}, \; x > 0, a, b > 0, \qquad (13)$$

with mean given by $E(X) = a \times b$ and variance given by $Var(X) = a \times b^2$. In this case, the distribution is asymmetric, positive or negative, depending on the shape parameter a. If a = 1, the distribution is reduced to an exponential distribution with mean value $\frac{1}{b}$.

If samples are taken from a population with the probability density function defined in (13), the sampling distribution is known. If $\bar{X}$ is the mean of a sample of dimension n taken from a population with distribution G(a, b), then using the moment generating function of $\bar{X}$, the distribution of sample means is G(na, b/n).

For the simulation, 200,000 samples of dimension 5 were generated under conditions corresponding to the situations described in the previous point, i.e., the quality follows a Gamma distribution with different values of the shape parameter a (2, 3, and 4) and with a scale parameter b equal to 1, obtaining distributions with different degrees of asymmetry and kurtosis. To assess the robustness of the results, a study of symmetry and kurtosis was carried out in comparison with the homologous values of the normal distribution, the results of which are presented in Table 1 and from which we can see that:

1) the distributions of the sample means are asymmetric, with a decrease in the asymmetry coefficient (γ_F, Bowley´s asymmetry coefficient) as the shape parameter (a) increases;
2) the weights of the tails (τ_F a weight that includes the extreme quantiles and quartiles are higher than the weights of the tails of the normal distribution;
3) the interquartile range (IIQ) and the range of variation (IV) increase as the shape parameter of the Gamma distribution increases.

F	σ_c	τ_F	γ_F	$\chi_{0.1\%}$	$\chi_{1\%}$	$\chi_{25\%}$	$\chi_{50\%}$	$\chi_{75\%}$	$\chi_{99\%}$	$\chi_{99.9\%}$	*IIQ*	*IV*
Normal	*1*	1,00	0,00	-1,401	-1,050	-0,304	-0,001	0,300	1,042	1,384	0,604	2,786
F	*(a, b)*	τ_F	γ_F	$\chi_{0.1\%}$	$\chi_{1\%}$	$\chi_{25\%}$	$\chi_{50\%}$	$\chi_{75\%}$	$\chi_{99\%}$	$\chi_{99.9\%}$	*IIQ*	*IV*
Gama	*(2, 1)*	1,01	0,07	0,586	0,826	1,549	1,939	2,388	3,771	4,569	0,838	3,982
	(3, 1)	1,01	0,06	1,165	1,510	2,450	2,933	3,476	5,128	6,037	1,027	4,872
	(4, 1)	1,00	0,05	1,799	2,216	3,362	3,932	4,568	6,389	7,369	1,205	5,571

Table 1: – Bowley´s asymmetric coefficient (γ_F), tail weight (τ_F), IIQ and IV and different quantiles for the different groups, with n = 5.

Source: Own elaboration

Table 2 shows the results of the parameters of the Gamma sampling distribution and the control limits for the mean chart, obtained when the probability of a Type I error is 0.0027. We can conclude that the control limits (obtained under the condition of probability symmetry) give clear indications of deviation from normality in all situations. It should be noted that the control limits in G (2,1) (see values of -L and L) correspond to the situation of maximum deviation, which validates the results presented for the asymmetry and tail weight coefficients.

Quality distribution	*Distribution by Sampling*						
G(a, b)	*G(na, b/n)*	E(X)	SD(X)	*-L*	*L*	*LIC*	*LSC*
G(2,1)	*G(10, 1/5)*	2	0,6325	-2,189	3,856	0,616	4,439
G(3,1)	*G(15, 1/5)*	3	0,7746	-2,316	3,721	0,206	4,882
G(4,1)	*G(20, 1/5)*	4	0,8944	-2,402	3,637	-0,148	5,253

Table 2: – Sampling distributions, parameters, L values, LIC and LSC, with different values of the shape parameter and n = 5.

Source: Own elaboration

In addition to the works already mentioned, we can highlight that in the work of Borror et al. (1999), the robustness of a dynamic sampling method of individual observations with a Shewhart chart and an EWMA chart was studied, considering that the quality had, on the one hand, a t-Student distribution with different degrees of freedom and, on the other hand, a Gamma distribution with different values of the shape parameter.

2.3. COMPARING METHOD PERFORMANCE WHEN QUALITY IS NORMAL DISTRIBUTED

The statistical, economic, or economic-statistical performance of control charts can be studied using different indicators. The most common in the literature is the AATS. In this study we will use the AATS to compare the efficiency of methods, assuming they are under the same control conditions. Consider the $AATS_{FSI}$ given in (10) and the $AATS_{VSI}$ given in (12) as well as the $AATS_{LSI}$ given in (8). Given that L = 3, d = 1 for the FSI method, K = 3,8134 for the LSI method, and that the quality characteristic has a normal distribution, the efficiency of the methods is compared using the ratio Q_N given by

$$Q_N = \left(\frac{AATS_{[MC]}}{AATS_{LSI}} - 1 \right) \times 100 \qquad (14)$$

where [MC] is the method being compared, replaced by FSI or VSI, and Q_N is a measure of the relative change in % of the AATS value when the [MC] method is used instead of the LSI method. It should be noted that although results are shown for changes in mean, standard deviation, and mean/standard deviation, only results for changes in mean are shown. The values of the Q_N ratio obtained by comparing the LSI and FSI effectiveness are shown in Figure 1:

1) In general, the average control chart with LSI is more efficient than the average control chart with FSI in the detection of small and moderate changes, i.e., in the detection of changes whose probability is not high; the maximum reduction in AATS obtained with LSI is significantly greater than the maximum reduction in AATS obtained with FSI;
2) For changes with a high probability of detection, FSI performs better than LSI, a situation in which the average number of samples until a signal (or false alarm) is very low, so that the interval between the moment the fault occurs and the moment the sample is taken after the fault is extremely important, equalizing the malfunction period whenever only one sample is needed to detect the change; in the case of an average sampling interval equal to 1, the average value of this interval, E(G), is equal to 0.61 in the LSI method and 0.5 in the FSI method;
3) About monotony, the values of the ratio initially increase, reach a maximum, and then decrease; the values of the change in the mean, λ, for which the ratio increases faster, are smaller the greater the sample size;

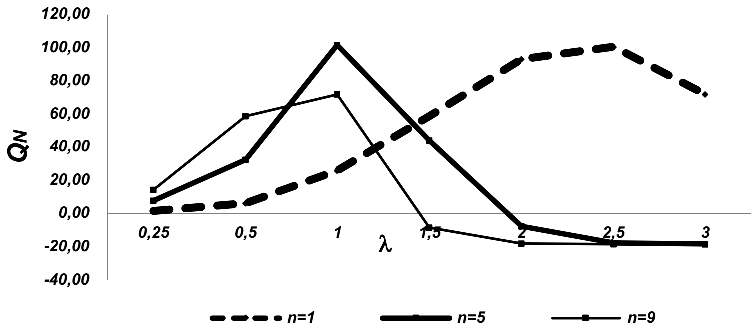

Figure 1: – Q_N values, %, with [MC] = FSI, as a function of λ, with ρ = 1, d = 1 in FSI and different sample sizes.

Source: Own elaboration

4) When individual observations are used, which is not usual, the LSI method is always more effective than the FSI method.

To compare the performance of LSI with VSI, four pairs of VSI sampling intervals are considered, considering the indications suggested in the works of Reynolds et al., (1988) and Amin & Miller, (1993), and considering d_1 values close to the extremes (minimum and maximum) of the LSI sampling intervals.

Considering these assumptions and that in VSI $E(D_0) = 1$, the efficiency of the VSI and LSI methods is compared using the ratio Q_N, given in (14), replacing [MC] by VSI. The results are shown in Table 3, from which we can conclude that:

1) If we consider a VSI sampling pair with a higher value in d_1, $d_1 = 0.5$, the performance of LSI is always better; since in LSI we obtain sampling intervals of less than 0.5, in this method more samples are taken, so the speed of detection increases;

2) In the situation referred to in 1), the ratio values increase with increasing λ until they reach a maximum, then they decrease and remain identical for the highest values of λ;
3) In the same situation, for small and moderate changes in the mean, the ratio values increase slightly as the largest interval decreases; for large changes in the mean, the ratio values decrease more sharply;
4) When we use d_1 = 0.1, and in the sample sizes most commonly used in the literature, n = 5, 9, the LSI method is faster than the VSI method only in detecting changes in the mean with magnitudes greater than 1.5, $\lambda \geq 1.5$, i.e., in situations where the probability of detection is high;
5) In the situation considered in 4), and if single observations are used, which is not very interesting in practical terms, the LSI method is faster than the VSI method only when $\lambda = 3$;
6) In general, the differences between the methods are more significant when d_2 = 2 in VSI;

n	***(d_1, d_2)*** / ***$AATS_{LSI}$***	***λ***						
		0,25	***0,5***	***1***	***1,5***	***2***	***2,5***	***3***
1	***(0.1, 2)***	-0,7	-3,0	-11,0	-19,3	-18,3	-0,5	24,1
	(0.5, 2)	0,2	0,7	4,1	13,2	29,1	43,1	44,8
	(0.1, 1.5)	-0,4	-1,8	-7,1	-14,1	-17,8	-11,8	0,5
	(0.5, 1.5)	0,3	1,1	5,4	14,5	27,6	35,6	31,1
	$AATS_{LSI}$	276,43	145,61	34,46	9,12	3,01	1,37	0,87
5	***(0.1, 2)***	-3,7	-13,2	-11,9	37,4	53,2	54,9	55,0
	(0.5, 2)	0,9	5,6	36,8	40,2	26,1	22,7	22,4
	(0.1, 1.5)	-2,3	-8,7	-16,3	7,9	17,2	18,3	18,3
	(0.5, 1.5)	1,4	7,0	32,7	23,6	6,1	2,3	2,0
	$AATS_{LSI}$	122,99	24,81	1,98	0,74	0,63	0,61	0,61
9	***(0.1, 2)***	-6,5	-19,3	24,1	53,3	55,0	55,0	55,0
	(0.5, 2)	1,9	13,2	44,8	25,9	22,5	22,4	22,4
	(0.1, 1.5)	-4,0	-14,1	0,5	17,3	18,3	18,3	18,3
	(0.5, 1.5)	2,8	14,5	31,1	5,9	2,1	2,0	2,0
	$AATS_{LSI}$	70,59	9,12	0,87	0,62	0,61	0,61	0,61

Table 3: – Q_N values, with [MC] = VSI, as a function of λ, with $\rho = 1$, different VSI sampling pairs and three sample dimensions.

Source: Own elaboration

7) In general, the greatest reductions are always obtained with the LSI method, much greater than those obtained with the VSI method; remember that, due to the value of the minimum and maximum sampling interval obtained with the LSI method, the sampling pair $(d_1, d_2) = (0.1, 2)$ is the one that brings the methods closest together.

This shows that in certain situations, with high sampling and failure costs, LSI can be a good alternative to the VSI method.

2.4. COMPARING METHOD PERFORMANCE WHEN QUALITY IS GAMMA DISTRIBUTED

Here we evaluate the effectiveness of the methods under non-normality of the quality characteristic. In addition to some of the algebraic expressions mentioned above, we also used simulation to obtain some of the results, generating 200,000 samples of dimension 5 using the Monte Carlo method and under the conditions described in the previous point. We compare the performance of the LSI method with that of the FSI and VSI methods when the quality characteristic X has a Gamma distribution and different values for the shape parameter, using the ratio Q_N, defined in (14). From the values of the Q_N ratio, when comparing the AATS of the LSI method with the AATS of the FSI method, Table 4, we can conclude that:

1) The effectiveness of LSI increases when we consider that the quality characteristic X has a Gamma distribution; the number of changes for which LSI is more effective than FSI increases;
2) The LSI method becomes less effective as the asymmetry and kurtosis coefficients of the Gamma distribution decrease, i.e., as the value of the shape parameter of the distribution increases;

3) The reductions obtained with LSI are still much greater than those obtained with FSI; in this situation, some of the reduction values are double those obtained with the normalized quality characteristic.

G(a,·b)	LSI	FSI	LSI	FSI	LSI	FSI
	G(2,·1)		G(3,·1)		G(4,·1)	
λ	AATS	$Q_{FSI,2}$	AATS	$Q_{FSI,3}$	AATS	$Q_{FSI,4}$
0	**370,05**	0,0	**370,03**	0,0	**370,03**	0,0
0,25	**233,56**	8,3	**217,71**	8,2	**206,84**	8,2
0,5	**67,32**	35,7	**59,65**	35,6	**54,81**	35,4
0,75	**18,51**	89,5	**15,85**	88,8	**14,26**	88,0
1	**5,32**	171,0	**4,50**	167,2	**4,05**	163,0
1,25	**1,88**	238,9	**1,63**	224,4	**1,50**	210,5
1,5	**0,97**	212,4	**0,88**	185,9	**0,85**	165,4
1,75	**0,72**	117,9	**0,69**	95,0	**0,68**	80,2
2	**0,64**	39,9	**0,63**	27,8	**0,63**	20,9
2,5	**0,62**	-15,7	**0,61**	-16,3	**0,61**	-16,6
3	**0,61**	-18,4	**0,61**	-18,4	**0,61**	-18,4

Table 4: – Values of $AATS_{LSI}$ and Q_N, as a function of λ,
for Gamma distribution with a = 2, 3 and 4, d = 1 in FSI and n = 5.
Source: Own elaboration

From the Q_N ratio values, when comparing the $AATS_{LSI}$ values with the $AATS_{FSI}$ values, in Table 5, it can be concluded that

1) The performance of LSI improves in all situations as the shape parameter of the Gamma distribution increases; the method loses effectiveness for small and moderate changes in the mean, and the LSI method gains effectiveness for large changes;
2) When $(d_1, d_2) = (0.1, 1.5)$ and $(d_1, d_2) = (0.1, 2)$ in VSI, the evolution of the Q_N ratio under normal conditions and Gamma distribution conditions is identical for the different magnitudes of changes in the process.

3) If we change the minimum sampling interval in VSI (Table 6) and increase its value to a value close to the minimum sampling interval obtained with LSI, LSI is better than VSI in all the situations considered, be it asymmetry or changes in the process mean;
4) Also, when $(d_1, d_2) = (0.5, 2.0)$ and $(d_1, d_2) = (0.5, 1.5)$ in VSI, the evolution of the Q_N ratio in normal and non-normal conditions is identical;

G(a, b)	LSI	VSI		LSI	VSI		LSI	VSI	
	G(2,1)			G(3,1)			G(4,1)		
(d_1, d_2)		(0.1, 2.0)	(0.1, 1.5)		(0.1, 2.0)	(0.1, 1.5)		(0.1, 2.0)	(0.1, 1.5)
(-w, w)		(-0.67, 0.57)	(-0.91, 0.897)		(-0.67, 0.58)	(-0.92, 0.898)		(-0.67, 0.58)	(-0.92, 0.90)
λ	AATS	$Q_{VSI.2}$		AATS	$Q_{VSI.3}$		AATS	$Q_{VSI.4}$	
0	**370,05**	0,1	0,0	**370,03**	0,1	0,0	**370,03**	0,1	0,0
0,25	**233,56**	4,4	4,8	**217,71**	3,0	3,7	**206,84**	2,2	3,0
0,5	**67,32**	-7,5	-0,7	**59,65**	-8,1	-1,5	**54,81**	-8,5	-2,1
0,75	**18,51**	-32,4	-17,1	**15,85**	-29,6	-16,2	**14,26**	-27,8	-15,6
1	**5,32**	-43,9	-33,7	**4,50**	-38,2	-29,5	**4,05**	-34,4	-27,0
1,25	**1,88**	-17,4	-26,0	**1,63**	-10,5	-20,2	**1,50**	-6,6	-17,0
1,5	**0,97**	24,3	1,1	**0,88**	30,4	5,2	**0,85**	32,5	6,4
1,75	**0,72**	47,4	16,0	**0,69**	50,6	17,8	**0,68**	51,0	17,7
2	**0,64**	53,8	18,9	**0,63**	55,1	19,5	**0,63**	55,0	19,3
2,5	**0,62**	54,8	18,2	**0,61**	55,2	18,5	**0,61**	55,2	18,5
3	**0,61**	55,0	18,3	**0,61**	55,0	18,3	**0,61**	55,0	18,3

Table 5: – Values of $AATS_{LSI}$ and Q_N, as a function of λ, for Gamma distribution with a = 2, 3 and 4, $(d_1, d_2) = (0.1, 1.5)$ and $(d_1, d_2) = (0.1, 2)$ in VSI and n = 5.

Source: Own elaboration

5) The values of the ratio maxima in Table 6 are approximately double the maxima in Table 5, showing the increased efficiency of LSI when the sampling intervals of the methods are close.

G(a,-b)	*LSI*	*VSI*		*LSI*	*VSI*		*LSI*	*VSI*	
	G(2,1)			*G(3,1)*			*G(4,1)*		
$(d_1,-d_2)$	---	*(0.5,-2.0)*	*(0.5,-1.5)*	---	*(0.5,-2.0)*	*(0.5,-1.5)*	---	*(0.5,-2.0)*	*(0.5,-1.5)*
(-w,-w)	---	*(-0.50,-0.35)*	*(-0.71,-0.61)*	---	*(-0.49,-0.36)*	*(-0.71,-0.62)*	---	*(-0.49,-0.37)*	*(-0.71,-0.63)*
λ	*AATS*	$Q_{VSI,2}$		*AATS*	$Q_{VSI,3}$		*AATS*	$Q_{VSI,4}$	
0	*370,05*	0,0	0,0	*370,03*	0,0	0,0	*370,03*	0,0	0,0
0,25	*233,56*	5,9	6,1	*217,71*	5,1	5,3	*206,84*	4,6	4,9
0,5	*67,32*	9,5	12,0	*59,65*	9,2	11,6	*54,81*	9,0	11,3
0,75	*18,51*	17,[illegible]	22,2	*15,85*	18,8	23,2	*14,26*	19,7	23,7
1	*5,32*	48,5	50,1	*4,50*	50,0	51,2	*4,05*	50,3	51,0
1,25	*1,88*	96,2	90,0	*1,63*	93,3	86,4	*1,50*	89,1	81,8
1,5	*0,97*	107,9	95,0	*0,88*	99,5	85,4	*0,85*	91,5	76,8
1,75	*0,72*	78,7	61,3	*0,69*	70,4	52,1	*0,68*	63,9	45,5
2	*0,64*	47,6	28,2	*0,63*	43,0	23,2	*0,63*	39,8	20,0
2,5	*0,62*	23,9	3,6	*0,61*	23,7	3,3	*0,61*	23,5	3,1
3	*0,61*	22,4	2,0	*0,61*	22,4	2,0	*0,61*	22,4	2,0

Table 6: – Values of $AATS_{LSI}$ and Q_N, as a function of λ, for Gamma distribution with a = 2, 3 and 4, $(d_1, d_2) = (0.5, 2.0)$ and $(d_1, d_2) = (0.5, 1.5)$ in VSI and n = 5.

Source: Own elaboration

From the results obtained, it can be concluded that the LSI method is very robust in different application scenarios and is even more efficient than the FSI and VSI sampling methods for any type of change in the process quality average.

3. EXAMPLE OF APPLICATION IN THE QUALITY CONTROL OF THE PRODUCTION OF AMMUNITION

Let's now look at an example of a practical application, in this case, a production process for medium caliber ammunition. Suppose that the average lifetime of the production system is 100 time units; that the cost of sampling is €1 per ammunition inspected; that the cost of system failure is €100/unit defective for small changes in the average and €1000/unit defective for large changes in the average (because the probability of the product not meeting the defined specifications is high and we may have to throw away the whole product). Let's also assume that the average sampling interval, under control, for the VSI and LSI methods is 1 and that n = 5. The average number of false alarms is 0.3 and the associated costs can be ignored. So, if there is a change in the average of 0.5 ($\lambda = 0.5$), an average of 33.4 samples are required to detect the change. The cost of sampling is €667 (100×5 (under control) + 33.4×5 (out of control)):

- If (d_1, d_2) = (0.1, 2) in VSI, the failure cost is 21.53×100 = 2153€;
- In the LSI method, the cost of the failure is 24.81×100 = €2481; in this situation, the cost per unit of time associated with using the LSI method is approximately 9% higher than the cost per unit of time associated with using the VSI method (23.20 vs. 25.22).

If there is an average change of magnitude 2 ($\lambda = 2$), an average of 1.08 samples are required to detect the change. The cost of sampling is €505.4 (100×5 (under control) + 1.08×5 (out of control)):

- If (d_1, d_2) = (0.1, 2) in VSI, the cost of the failure is 0.95×1000 = 950€;
- In the LSI method, the cost of the failure is 0.63×1000 = €630; in this situation, the cost per unit of time associated with using the LSI method is approximately 22% lower than the cost per unit of time associated with using the VSI method (14.42 vs. 11.28).

Naturally, we can conclude that in situations where any kind of change in the average can occur (small, moderate, or large), the monetary gains obtained by using LSI can be significantly higher than those obtained by using the VSI method.

4. CONCLUSIONS

In the context of the restrictions of the study, where we can emphasize those, we had with obtaining the costs related to the production of ammunition, we can conclude that the proposed sampling method performs very well in detecting changes in the mean of the quality characteristic under study, compared to the performance of the fixed method and the VSI method in a context of normal distribution.

If we compare the methods under the condition that the quality characteristic has a Gamma distribution, with different degrees of non-symmetry, the performance of the LSI method generally remains the same or improves.

From a practical point of view, the use of LSI can lead to savings in the order of 22% of ammunition production costs compared to the use of VSI.

Therefore, in the future we intend to consider the method for simultaneous charts (mean and standard deviation), EWMA charts, CUSUM charts, and charts for single observations, situations where we believe the method could improve its performance.

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