

**THE PRODUCTION OF PEDAGOGIC NARRATIVES AS A FORMATIVE
PROCESS IN CONTEXTS OF COLLABORATION**

ADAIR NACARATO

Programa de Pós-Graduação Stricto Sensu em Educação, Universidade São Francisco, Itatiba/SP, Brazil
ada.nacarato@gmail.com | <https://orcid.org/0000-0001-6724-2125>

IRIS APARECIDA CUSTÓDIO

Colégio Bom Jesus – Unidade Itatiba/SP, Brazil
irisapcustodio@gmail.com | <https://orcid.org/0000-0003-3149-240X>

ROSANGELA BERTOLDO FRARE

E. E. Monsenhor Honório Heinrich Bernard Nacke/SEE-SP, Morungaba, Brazil
robertoldo81@hotmail.com | <https://orcid.org/0000-0002-5009-2981>

ABSTRACT

This text aims to present evidence of how collaboration constitutes an educational practice and facilitates the professional development of mathematics teachers. For this purpose, it discusses fragments of the narrative of a teacher who participates in a collaborative group and works in a Brazilian public school with high school students (14 to 17-year-olds). We debate the Vygotskian and Bakhtinian concepts of collaboration as practices of education and the role of pedagogic narratives as devices of teacher education. The analysis shows the extent to which the teacher appropriated the principles defended in the group, of a social culture in mathematics classes, as well as the discourses circulating in it, acted as a co-builder of mathematical knowledge with her students, exerted agency, and developed professionally.

KEYWORDS

collaboration; professional development; mathematical discourse.



SISYPHUS

JOURNAL OF EDUCATION

VOLUME 9, ISSUE 02,

2021, PP.11-33

DOI: <https://doi.org/10.25749/sis.21782>

CC BY-NC 4.0

A PRODUÇÃO DE NARRATIVAS PEDAGÓGICAS COMO PROCESSO FORMATIVO EM CONTEXTOS DE COLABORAÇÃO

ADAIR NACARATO

Programa de Pós-Graduação Stricto Sensu em Educação, Universidade São Francisco, Itatiba/SP, Brasil
ada.nacarato@gmail.com | <https://orcid.org/0000-0001-6724-2125>

IRIS APARECIDA CUSTÓDIO

Colégio Bom Jesus – Unidade Itatiba/SP, Brasil
irisapcustodio@gmail.com | <https://orcid.org/0000-0003-3149-240X>

ROSANGELA BERTOLDO FRARE

E. E. Monsenhor Honório Heinrich Bernard Nacke/SEE-SP, Morungaba, Brasil
robertoldo81@hotmail.com | <https://orcid.org/0000-0002-5009-2981>

RESUMO

Este texto tem como objetivo apresentar indícios de como a colaboração se constitui em prática de formação e possibilita o desenvolvimento profissional do professor que ensina matemática, a partir da análise de excertos da narrativa de uma professora participante de um grupo colaborativo. Trata-se de uma professora da educação básica, que atua numa escola pública brasileira, com estudantes do ensino médio (14-17 anos). Discute-se a concepção de colaboração como prática de formação a partir dos estudos Vigotskianos e Bakhtinianos e o papel das narrativas pedagógicas como dispositivos de formação docente. A análise traz evidências do quanto a professora se apropriou dos princípios da cultura social de sala de aula de matemática defendidos no grupo e dos discursos que nele circulam, atuou na coconstrução de conhecimentos matemáticos com seus alunos, exerceu sua agência e se desenvolveu profissionalmente.

PALAVRAS-CHAVE

colaboração; desenvolvimento profissional; discurso matemático.



SISYPHUS

JOURNAL OF EDUCATION

VOLUME 9, ISSUE 02,

2021, PP.11-33

DOI: <https://doi.org/10.25749/sis.21782>

CC BY-NC 4.0

LA PRODUCCIÓN DE NARRATIVAS PEDAGÓGICAS COMO PROCESO FORMATIVO EN CONTEXTOS DE COLABORACIÓN

ADAIR NACARATO

Programa de Pós-Graduação Stricto Sensu em Educação, Universidade São Francisco, Itatiba/SP, Brasil
ada.nacarato@gmail.com| <https://orcid.org/0000-0001-6724-2125>

IRIS APARECIDA CUSTÓDIO

Colégio Bom Jesus – Unidade Itatiba/SP, Brasil
irisapcustodio@gmail.com| <https://orcid.org/0000-0003-3149-240X>

ROSANGELA BERTOLDO FRARE

E.E. Monsenhor Honório Heinrich Bernard Nacke/SEE-SP, Morungaba, Brasil
robertoldo81@hotmail.com| <https://orcid.org/0000-0002-5009-2981>

RESUMEN

Este texto tiene como objetivo presentar evidencias de cómo la colaboración se constituye en la práctica formativa y posibilita el desarrollo profesional del docente que imparte matemáticas. Parte del análisis de extractos de la narrativa de una docente de educación básica participante en un grupo colaborativo y que trabaja en una escuela pública brasileña con estudiantes de educación media (14-17 años). El concepto de colaboración como práctica de formación se discute a partir de los estudios Vigotskianos y Bakhtinianos y el papel de las narrativas pedagógicas como dispositivos de formación docente. El análisis aporta evidencia de cuánto la docente se apropió de los principios de la cultura social de la clase de matemáticas defendidos en el grupo y los discursos que circulan en él; actuó en la co-construcción del conocimiento matemático con sus alumnos, ejerció su agencia y se desarrolló profesionalmente.

PALABRAS - CLAVE

colaboración; desarrollo profesional; discurso matemático.



SISYPHUS

JOURNAL OF EDUCATION

VOLUME 9, ISSUE 02,

2021, PP.11-33

DOI: <https://doi.org/10.25749/sis.21782>

CC BY-NC 4.0

The Production of Pedagogic Narratives as a Formative Process in Contexts of Collaboration¹

Adair Nacarato², Iris Aparecida Custódio, Rosângela Bertoldo Frare

INTRODUCTION

Since the 1990s, researchers of Mathematics Education have appropriated theoretical constructs such as collaborative practices, collaborative work, or collaborative group, in their investigations on teacher education. Many researchers contributed to the concept of collaboration, as can be seen, for example, in the book *Reflecting and investigating professional practice [Refletir e investigar sobre a prática profissional]*, published in 2002 by the Working Group on Investigation (GTI), a milestone for many researchers that came later – especially the text of Boavida and Ponte (2002). In that text, the authors distinguish cooperation and collaboration by applying the debates of Day (1999, as cited in Boavida & Ponte, 2002, p. 45), in which it is stated: “while in cooperative works the participants’ power relations and roles are not called into question, collaboration involves careful negotiations, joint decision-making, effective communication, and mutual learning”. Collaboration (co-labouring) implies working together, interaction, and sharing. The work of Andy Hargreaves (1998) was equally important for the present debate, differentiating spontaneous collaboration (which occurs by the participants’ initiatives) from artificial collaboration (imposed by superior instances).

These ideas contributed to a broader discussion on the notions of collaborative work and group. Fiorentini (2004), for example, tracks the incidence of terms related to collaboration in Brazilian research and seeks to establish the characteristics and constitutive elements of collaborative work. The author highlights wilfulness, identity, and spontaneity, shared leadership or co-responsibility, mutual support, and respect as central aspects of such work.

Another important reference for our research is Passos et al. (2006), whose meta-analysis of Brazilian studies on the professional development of mathematics teachers pointed to the importance of collaborative practices as catalysers of teachers’ professional development.

Yet another work that gained relevance and was well spread within the community was Ibiapina’s (2008). Seeking to deepen the debates on collaborative research methodologies, the author presents the results of a bibliographic review that situates the beginning of collaborative research in the 1980s, along with emancipatory practices and professional development. For Ibiapina (2008), collaborative research is a continuous practice of teacher education, and collaboration happens among peers in dialogical relations of social interaction. Researching collaboratively means working in groups marked by reflexive processes that support participants in their analyses of different problems affecting the school routine. Hence, it requires a dialectical movement between theory and practice that enables the production of meanings about teaching practices.

1 Research financed by CNPq, Pesquisador Produtividade.

2 Rua Antonio Benedito Casarin, 535. Itatiba/SP. CEP 13.252-743, Brasil.



These ideas emerged and were amplified in several investigative works. Coelho (2017) reviewed the theses and dissertations focusing on groups that were published from 2002 to 2012, in Brazil. The author identified 54 works and dwelt on six that centred on collaborative groups. She argues for the relevance of heterogeneous groups which allow multiple gazes and points of view, as well as of theoretical studies, while articulating the potentialities of collaborative groups for teachers' professional development.

Another referential work was the survey of Robutti et al. (2016). It focused on teachers involved in collaboration in formal spaces of professional development and other collaborative environments. According to the authors, works on these subjects rarely theorised collaboration, with only a few reporting on how the adopted theoretical frameworks shaped the research design, and it is also difficult to relate teachers' learning with collaboration in a project. They found many studies on the collaborative work of teachers in the first and final grades, but only a few centred on teachers of both segments working and learning together. When centred on that aspect of the teaching work, however, conclusions showed to what extent the teachers learned together in terms of comprehending students' progression from one grade to the other. The authors also stressed that an even smaller number of works analysed teachers' mathematical learning to identify what they needed to learn in order to teach.

Focused on professional development, some works show evidence of how collaboration contributes to teacher's learning and teaching practices "with their individual, social, and occupational dimensions in a collaborative, inquisitive, and a self-directed learning environment" (Sancar, Atal, & Deryakulu, 2021, p. 4). Proposing a new theoretical framework for teachers' professional development, the authors argue that it "should provide teachers the opportunity to practice collaboratively and to reflect and gain feedback from each other" (p.6). Gathering the voices of teachers involved in a professional development program, Haug and Mork (2021, p. 2) point to five central characteristics of the professional development effectivity: "(1) content focus; (2) active learning; (3) collective participation; (4) coherence; and (5) duration".

For the most part, the results of these studies show that collaboration contributes to professional development; thus, in the next section, we will debate the entanglement of these two constructs. We draw on the affirmation by Robutti et al. (2016), that research that analyses how the work methodology built in a collaborative group potentialized teachers' learning and professional development is still much needed. That was the motivation of the present text. We participate in a collaborative group constituted by teachers who work in different levels of basic education and that systematises its investigations in the classroom by means of pedagogic narratives. The group employs the design research methodology. Syntheses of our research show how much the teachers learned and show their professional development derived from the collaboration existing in the group. Thus, our goal in the present text is to present evidence of how collaboration constitutes a practice of education and professional development for the mathematics teacher, in this group. For that purpose, we will use excerpts of the narrative of a teacher who participates in the group.

The present text is organised in three sections. In the first one, we debate our conception of collaboration as a formative practice within Vygotskian and Bakhtinian studies and introduce the collaborative group investigated in this research and its work methodology. Afterwards, we debate the role of pedagogic narratives as devices of teacher



education. Finally, we analyse the pedagogic narrative of one of the group's teachers who is also one of the authors of the present text and we present our conclusions.

COLLABORATION AS A FORMATIVE PROCESS

Created in 2003, the Collaborative Group in Mathematics (*Grupo Colaborativo em Matemática*, Grucomat) is linked to the *Stricto Sensu* Graduate Program in Education of the São Francisco University, in the state of São Paulo. Its participants are mathematics teachers of public and private schools, teaching all levels from basic education to high school (14 to 17-year-olds), as well as in university. The group meetings are carried out every 15 days.

In the group, collaboration is related to the interactions among the participants and their diverse knowledge. The way such interactions occur allows all theoretical, methodological, and experiential baggage gathered in various formative and professional spaces to be the basis for re-signifying the mathematical contents and the teaching practices. Beyond that, we defend that a greater collaborative potential is linked to the reduction of power and oppression. As relations of trust and respect grow stronger, processes of knowledge production are also improved. The pleasure of sharing spaces voluntarily with other teachers with similar objectives and interests is also a relevant element of the environment – partaking and reflecting on practices and knowledge.

Such relations foster interpersonal and intrapersonal reflections based on the senses and meanings the group sees as necessary (Ibiapina, 2008). Reflections may be triggered by theoretical texts which bring previous knowledges into the discussion and mobilise or promote their re-elaboration. They may also stem from pedagogic narratives produced by the participants or from the analysis of videos or audio records of the practices in the classroom; from the analysis of students' registers; or, yet, from the speech of one of the members, as "enunciating makes thought accessible to social processes – that is, to collaboration" (Ibiapina, 2008, p. 49). In Bakhtinian terms, alterity causes discourses to affect and be affected. Thus, teachers appropriate their colleagues' practices and speeches, resulting in a practice and discourse that are characteristic of the group. Each one transforms these elements according to his or her own purposes.

There is, however, what Smolka (2000) defines as a *common place*, which makes these practices and discourses specific to that group in particular. Such knowledge – a *knowledge-of-practice* (Cochran-Smith & Lytle, 1999) – is elaborated and systematised by the teacher locally, generating new knowledge regarding practices of teaching and learning mathematics. By looking reflexively at his or her practice and systematising it, for instance, by means of a pedagogic narrative, the teacher builds an investigative position and no longer only consumes theories.

The process described drives professional development. Our conception of that development is built closely to Vygostkian conceptions of human development, understood as connected to the cultural realm and constituting a social and interpersonal happening that fosters the internalisation of cultural forms of behaviour which include the transformation of pedagogic activity. This process is based on the transformation of an activity that is primarily external into an internal and sign-mediated activity, and that has the word as its sign *par excellence* (Vygotski, 2007). Besides, "knowledge transmitted



by collaborative processes is essentially formative, creating zones of potential development” (Ibiapina, 2008, p. 50).

We also defend that “it is not the individual who explains *a priori* his or her way of relating with others, but the social relations in which he or she is involved that explain his or her ways of being, acting, thinking, and relating” (Smolka, 2000, p. 30, original emphasis). In these relations, the teacher grabs his or her professional agency. We agree with Oliveira and Cyrino (2011) that professional agency is to be observed in and along the trajectory and social contexts in which the teacher participates. In these contexts, he or she appropriates the ways for exercising the teaching practice and situates themselves in proximity to a political commitment in favour of the students’ learning.

The context in which the person acts mediated the action: “agency is always mediated by the interaction between the individual (attributes and inclinations), and the tools and structures of a social setting” (Lasky, 2005, p. 900), in such a way that it makes sense to talk about “mediated agency”. (Oliveira & Cyrino, 2011, p. 114, original emphasis)

For that reason, professional development includes interpersonal and intrapersonal relations that, however distinct they may be, are inseparable. Thus, professional development is bound by these relations and connected with learning in an alteritarian fashion.

Grucomat’s research focuses on basic education classrooms and its methodology includes: selecting a theme for investigation; carrying out theoretical and conceptual studies on the thematic; elaborating tasks for multi-level classrooms; the development of these tasks by the participant teachers and in partnership with other teachers in the schools where they work; audio or video records and collecting students’ productions during the development of tasks; the narrative written report of the class by the responsible teacher; socialisation and analysis of the records produced in the group; the re-writing of the tasks, if needed; and the renewed development in other teachers’ classrooms. The cycle repeats itself up to the point where the group considers the task adequate, tentatively, as classrooms are unpredictable and unique and what works in one of them might not work so well in another.

This methodology approaches the Design Research or Design-Based Research framework, seeking to integrate practice and research, focusing on learning, and showing that research and instruction designs are interdependent (Molina, Castro, & Castro, 2007). For these authors, designing learning situations constitutes the context for research which, in turn, improves practices. It is a process of production of theories and significant knowledge for practice; it is decisively oriented towards practice (Powell & Ali, 2018).

Theory producers who debate this methodology (Anderson & Shattuck, 2012; Molina et al., 2007; Powell & Ali, 2018) indicate its most important characteristics, out of which a couple are more directly related with Grucomat’s work methodology. Firstly, Design Research (DR) is related to a real educational context, involving instructions and investigations carried out in practice. That adds to the validity of the research, which may evaluate, inform, and improve not only the teacher’s direct practice, but also that of other teachers. It allows both the teacher’s and students’ learning, which are analysed in



the group, where we look into the students' records and mathematical discourse³ and how they present evidence of conceptual elaboration, as well as the teachers' pedagogic narratives, seeking to identify learning.

The DR methodology is intervention-oriented (Anderson & Shattuck, 2012; Powell & Ali, 2018) and it requires that the teacher-researcher be careful with the documents produced during the tasks. Such records allow the analysis of the potentialities and the reach of the task. Moreover, by adopting the cultural-historical perspective, we assume that classroom processes require pedagogic intentionality. Thus, the teacher who intends to develop the task prepares the needed documentation and creates a learning environment for his or her students. Such an environment is common in the workplaces of the teachers involved in the group, who seek a social culture of mathematics classes (Hiebert et al., 1997). The goal is a culture which privileges dialogue and group work, enabling interactions between students and the interventions adequate to the teacher.

DR assumes the involvement of all different participants (Molina et al., 2007); all of them are considered researchers and each contributes with their own experiences. The group is heterogeneous and composed by teachers who work in different levels, but who are united by collaboration. The group's heterogeneity enables reciprocal learning among teachers involved in different school levels.

DR produces knowledge, generates theories, and is generative (Powell & Ali, 2018). In the group, theory is our starting point, but it is revisited and re-signified from the perspective of the works developed in the classroom, within the contributions that students bring to the analysis of the proposed tasks. Students' mathematical discourses are fundamental for the theoretical construction and conceptual elaboration of the thematic in question. Knowledge is also produced about the teachers' acting expertise, how they attract students to the conversation, their interventions and analyses of the mathematical discourses circulating among students. According to Molina et al. (2007), DR fosters the development of constructs useful to identify regularities and standards in the complex contexts in which it is applied. It is prospective and reflexive (Powell & Ali, 2018), as carrying out tasks, a first moment of theorising by the teacher, allows a prospective look at the task which incites re-writings and adaptations. Finally, DR is iterative and ecologically valid (Powell & Ali, 2018). Its cycles comprise the study-elaboration of the task-classroom development-records-teacher and group analysis-rewriting of the task, which are reflexively repeated. Tasks are validated locally, in a micro cycle when the teacher points to her or his own frailties, and in a macro one, when the group discusses the potentialities of that experience, and the tasks are developed in other classrooms. Knowledge is acquired iteratively and retrospectively.

Molina et al. (2007) point to two types of analysis within DR – a preliminary one after each cycle and one at the end of the whole process, after a long period of data production. At the end of a study and its multiple cycles, the participants of the group systematise its productions into narratives and make them public. Besides contributing to the professional development of the involved, such productions are made available for a larger population of mathematics teachers in book chapters, articles in journals, and presentations in events. This is the political and ethical compromise the group holds of socialising the knowledge produced and assuring the visibility of the teachers' work. Thus, Grucomat is a community of investigation (Jaworski, 2008).

³ As a mathematical discourse, we understand any enunciation expressing mathematical thoughts. This conception will be further discussed in section 3.1.

This methodological approach is only possible due to collaborative work. In our views, only in a group of such nature can the iterative cycles of DR be carried out, since they demand a reflexive look upon each colleague's production with their students. In the interaction of cycles, there are processes of professional learning and development (Bannan-Ritland, 2008). This author names this process Teacher Design Research and understands it as a new approach, a paradigm for teachers' professional development, given that, when involved in the cycle of DR, it can promote learning and place the cycle itself in processes of reflection regarding its beliefs and practices.

Teachers share related theory and research, as well as participate in multiple cycles of data collection and analysis about students and teachers' learning. These cycles prompt design ideas that are grounded in theory, research findings, and the teachers' own practice. The teachers participate fully in decisions about the design, along with the research team and other interested parties, and about using the innovations in their own and their colleagues' classrooms. (p. 248)

This methodology also requires systematising the work developed in the classroom. We have been using classroom narratives – the pedagogic narratives – as a device for the systematisation of teachers' practices and, therefore, as a product of their research.

PEDAGOGIC NARRATIVES AS A TOOL FOR PROFESSIONAL DEVELOPMENT

The group has an archive of narrative documentations by its participants, a few of which have been selected by the participants and published as papers or book chapters. For the present article, we purposefully selected the narrative of Rosangela⁴, a teacher and the third author of the present text. She teaches mathematics for high-school students in a public state school and has participated in the group's activities since 2014. In previous occasions, she sent her narratives to be published in an e-book produced by Grucomat, showing how much she has appropriated the group's collective identity, which includes: debates; carefully planning the task so it is both investigative and challenging for students; an analytic look at classes; considering the actual classroom, with its level of unpredictability, especially when faced with an investigative task; and listening to students and learning from them.

The selected narrative focused on the development of tasks involving algebraic thought in her classrooms. The tasks were applied to two classes in the second year of high school, both from a public school where she teaches mathematics. Two tasks were developed (Figures 1 and 2), both adapted from the bank of questions of the Brazilian Mathematics Olympics in Public Schools (*Olimpiada Brasileira de Matemática das Escolas*

4 Available at: <http://www.sbembrasil.org.br/sbembrasil/index.php/publicacoes/colecao-sbem>.



Públicas, Obmep), and aimed at verifying if students noticed relations and regularities and were able to make generalisations using algebra.

A worm always moves along a straight line. Every day, it moves 5 meters forward and 3 meters backwards.

- After 15 days, how far will the worm be from the starting point?
- Is there any regularity in the sequence formed by the distances reached by the worm in relation to the starting point at the end of each day?
- Can you discover how far it will be from the starting point in any number of days? How?
- After 27 days, how many meters will the worm have travelled, in total?
- Is there any regularity in the sequence formed by the number of meters travelled by the worm at the end of each day?
- Can you discover how many meters it will have travelled at the end of any number of days? How?

Figure 1. Task 1 (adapted from Abreu et al., 2014, p. 51).

The sequence of figures below represents walls that are built using cubes:

Figure 1 Figure 2 Figure 3

- How many cubes are necessary to build a wall with 10 peaks?
- Is there any relation between the number of peaks in a wall and the number of cubes needed to build this wall?
- Can you create a mathematical expression to calculate the number of cubes needed for the construction of a wall with n peaks?
- How many peaks would there be in the wall represented by Figure 12?
- Is there any relation between the number of the figure and the number of peaks in the wall it represents?
- Can you create a mathematical expression to calculate the number of cubes needed for the construction of a wall with n peaks?
- How many cubes are needed to build the wall represented by Figure 16?
- Is there any relation between the number of the figure and the number of cubes necessary to build the wall it represents?
- Can you create a mathematical expression to calculate the number of cubes needed for the construction of the wall represented by figure N ?

Figure 2. Task 2 (adapted from Abreu et al., 2014, p. 26).

The tasks seek to foster a multiplicity of looks by the students, as well as their engagement in mathematical discussions, corroborating the perspective of Powell (2019):

Given our dialogic view of what it means to perform mathematics, tasks should engage learners in discursive interactions. The goal of the interactions is supporting their variant and invariant relations between mathematical objects. The tasks should lead learners to use their consciousness to observe, in the midst of mathematical objects, what is (logically) necessary about relations and about the relations of relations among objects. (pp. 192-193)

In the chosen narrative⁵, the teacher begins by presenting the two selected tasks and the classroom context, and by describing the development of the tasks. Then, she analyses the students' records for Task 1, which she sees as focused on mobilizing algebraic thought, and Task 2, which focuses on the variety of resolution strategies employed, in the conversations about laws of equivalent formation, and in the appearance of a relation that had not been suggested. She finishes with her considerations on the experience narrated.

Seeking to reach our goal – to show how collaboration constitutes a formation practice and enables the teacher's professional development by analysing the narrative of one of the group's teachers –, we selected excerpts that point to the oral and written mathematical discourse during the tasks in classroom, as well as to the teacher's professional development. We thus organised two thematic units, one involving the teacher's attention to the students' mathematical discourses and the other approaching her own professional development.

THE TEACHER'S REGARD TO STUDENTS' MATHEMATICAL DISCOURSE

We begin by making our conception of mathematical discourse explicit. We agree with Bakhtin (2011, p. 261) that "all the diverse fields of human activity are related to the use of language" and that we communicate using oral or written formulations that express specific conditions of production. The author employs the concept of "genres of discourse" to refer to the group of formulations specific to a precise field of communication. Such genres circulate in the different spheres of human activity and, in the case of the school, we consider the mathematical discourses circulating in the classroom. The author also considers the existence of a multiplicity of heterogeneous genres; however, he separates primary genres, those which are formed "in the conditions of immediate discursive communication" (Bakhtin, 2011, p. 263), from secondary genres, which are more complex (literature, scientific research, among others) and predominantly written.

In this text, we are concerned with the genres of mathematical discourse, considering their production in the classroom, where students and teachers communicate orally or in writing. The mathematical oral formulations of the students, though based on previously acquired concepts, may be initially considered primary genres, but, as they are collaboratively negotiated and re-signified in the classroom, they become secondary genres and are impregnated with mathematical discourse. Thus, as a discursive genre, the mathematical discourse is built in the dynamics of classroom communication and it is fundamental for the construction of mathematical thought itself; students not only express their own mathematical ideas, but also appropriate the ideas, reasonings, and ways of thinking of their colleagues. In this sense, the cultural-historical perspective contributes to our understanding of the modes of production of mathematical knowledge within the circulation of formulations in texts and classroom discourses by considering the intrinsic relationship between thought and language.

⁵ Given the length of the narrative, it could not be included in full in the present paper.



Sfard (2001) contributed to our reflections as this author also employs Vygotskian studies to analyse mathematical discourses as the condition for learning with comprehension. Communication is central for the learning process: “Thinking arises as a modified private version of interpersonal communication. All this amounts to the claim that thinking is nothing but our communicating with ourselves, not necessarily inner, and not necessarily verbal” (Sfard, 2001, p. 26). By communicating, the students expose the process of organising thought and articulating previous knowledge with the new information that circulated in the multiple formulations in the classroom – formulations previously appropriated during the literacy process or that are currently appropriated, understood as the “interiorisation of higher mental functions” by their transmission from the “inter-psychological” to the “intra-psychological plane” (Sfard, 2001, p. 23). Thus, the formulations (vocabulary, properties, symbolic language) of mathematical discourses are shared in the dialogical dynamics of the classroom; however, thought has no independent existence in enunciation; it articulates with other formulations and texts.

High sensitivity of our ways of acting to social, cultural, historical and situational contexts is an inevitable derivative of the fact that the activities themselves, rather than being dictated by an external non-human world, have their roots in our cultural heritage and are constantly shaped and re-shaped by successive generations of practitioners. (Sfard, 2001, p. 25)

A dialogical practice of teaching mathematics, based on interpersonal interactions and communication, enables the appropriation of mathematical concepts with comprehension.

A teacher who defends this perspective of teaching mathematics not only provides students with tasks that contribute to their discussion and communication, but also demonstrates an analytic look upon the discourses of the students, and his or her interventions contribute towards advances in learning. Such a look occurs amid classroom communication and when analysing the students’ production. The excerpts show this double action by teacher Rosangela, in her interaction and negotiation of meanings in the classroom and in analysing the students’ records, producing the narrative.

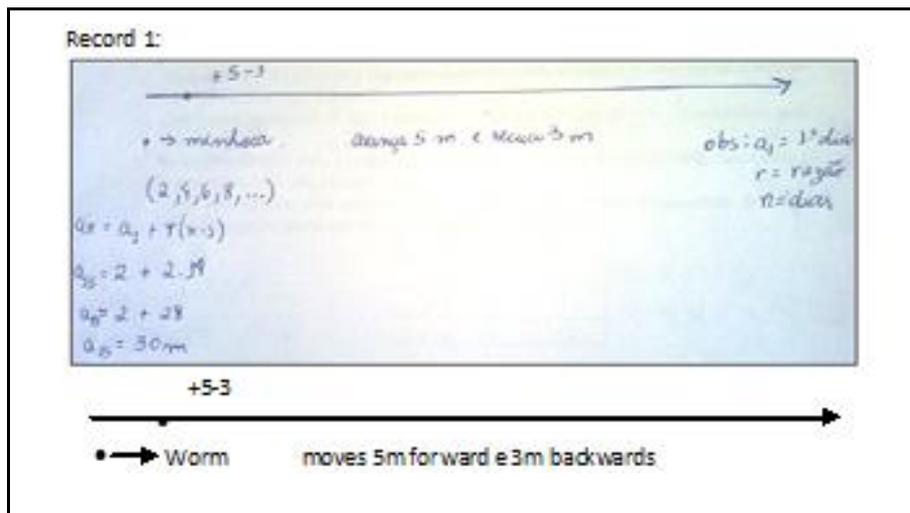
The teacher begins her narrative reporting the development of the tasks. She required that students were organised in freely chosen groups, respecting the limit of four students per group. Each group received copies of the tasks with blank spaces to register the solutions. Five classes were necessary for each classroom to complete the process, including the production of two tasks and socialisation. While students solved the tasks, the teacher circulated around the groups, encouraging students to compose written records, observing these records, and inciting students to notice regularities and make generalisations, so as to further develop algebraic thought.

Narrating the development of tasks, the teacher analyses the students’ mathematical discourse in their records. The first issue she highlights is the variety of strategies to solve Task 1, which was proposed so that students “*not only became familiar with the task type and classroom organisation, but also appropriated the ways of registering thought and making generalisations, even as related to simpler*

situations”⁶(Frare, 2019, p. 164). As she herself reports, students were not used to investigative classes and to collaborative work in groups; thus, at first, they were destabilised by these practices:

As I distributed the sheets, they wanted to know the “operation” they would be asked to do and what exactly they should write in the sheet. This is because Maths classrooms are traditionally based on the “exercise paradigm” (Alro & Skovsmose, 2006). (...) I knew the writing exercise was not going to be easy, as it breaks with a culture of the Maths classes. (Frare, 2019, p. 163)

She begins by introducing us to a classroom reality and to how little experience students have with more investigative classes. Afterwards, she analyses the mathematical discourse in students’ records. To identify how students determine the distance from the worm’s original position to its position within 15 days, in Task 1, she brings their different records and interprets them, demonstrating the mathematical discourse formed in that context. Excerpts of these records selected by the teacher in her narrative are presented in Figure 3.



6 The teacher’s narrative will be written in italics.



(2, 4, 6, 8, ...)

$$a_n = a_1 + r(n-1)$$

$$a_{15} = 2 + 2 \cdot 14$$

$$a_{15} = 2 + 28$$

$$a_{15} = 30m$$

obs: $a_1 = 1^{st}$ day
 $r =$ ratio
 $n =$ days

Record 2:

a) (2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30)
 R: Distância a distância de 30 m.

a) (2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 24, 26, 28, 30)
 R: It will be at a 30m distance.

Record 3:

Handwritten calculations showing a sequence of numbers and the result 30m.

a) moves forward backwards

1 st	5m	3m
	75m	45m
15	15	
<u>X5</u>	<u>X3</u>	
75	45	
	<u>-45</u>	
	30m	

Record 4:

Handwritten calculations showing a sequence of numbers and the result 30m.

1 day = 5m - 3m
 1 day = 2m

1 day = 2m
 15 days = ?

15 days = 2 · 15
 15 days = 30m

Figure 3. Different strategies employed by students in question a, Task 1 (Frare, 2019, pp. 164-165).

Record 1 captures the reader's attention as it shows students making connections between something new, the current task they develop, and something they had already studied the year before – the number sequences (Arithmetic and Geometric Progressions – AP and GP) –, in a process of signification: "when observing the distance in meters between the worm's starting point and its progress each day, the group observed it was

an AP and its ratio was 2. Thus, they employed the formula for the generalised sum to determine the distance on the 15th day” (Frare, 2019, p. 164).

The other records (2, 3, and 4) highlight that, whether the previously mentioned relation has been established or not, the groups employed diversified strategies in order to solve question *a*, demonstrating the large number of formulations a class presents when facing an investigative task. Regarding record 2, the teacher affirms that “*after observing that the worm was distanced by 2 meters more from the starting point than on the previous day, one of the groups decided to compose a sequence with the distances from the 1st to the 15th day*” (Frare, 2019, p. 164).

Analysing the mathematical discourse in records 3 and 4, the teacher notices that students employed proportionality to solve the problem. She affirms that “*there have been answers involving the distance forward and backwards, as well as only the distance of the worm at the starting point*” (Frare, 2019, p. 165). In record 3 she states: “*knowing that the worm moved 5 meters forward and 3 meters backwards every day, the group obtained the total amount of meters it would move forwards and backwards in 15 days and subtracted one from the other, finding how distant the worm would be from the starting point at the end of the period*” (Frare, 2019, p. 165). Finally, in record 4, she indicated that students realised that “*the worm advanced only 2m from the starting point every day, calculated 2 times 15, and obtained the answer*”.

These mathematical discourses provide hints on students’ processes of developing of algebraic thought as they demonstrate the relations they do or do not establish by using the varied strategies. She dwells on each one of these discourses, seeking to interpret them from a mathematical perspective. Another relevant point of the narrative refers to the mathematical discourses that circulate during the socialisation of tasks, in a dialogue referring to the different laws of formation raised during the resolution of question *c* in Task 2 and the perception of equivalences among them. We exemplify this by showing an excerpt of the dialogue between teacher Rosangela and her students (Frare, 2019, p. 171):

Rô: Were you able to create a mathematical expression to calculate the number of cubes in a wall with n peaks?

Alexia: We wrote a_n , which is the number of cubes, the ratio, times the number of peaks, subtract 1, which is $a_n = r \cdot n - 1$.

Rô: Let’s see if we can calculate the number of cubes. How much is the ratio? What is the ratio?

Luana: It’s 3. It represents by how much the number increases. Then, it’s $a_n = 3 \cdot n - 1$.

Rô: Let’s put the number 3 here. The number of peaks is the “ n ”. How many peaks are there in the first figure?

Students: 2.

Rô: So, it is 3 times 2 minus 1, which is...

Students: 5.

Rô: It’s 5 and there actually are 5 cubes in the first. Let’s test it with the second. 3 times 3 minus 1 is 8, and there are 8 cubes. Do you think this expression is good to calculate the number of cubes according to the number of peaks?

Students: Yes.



Rô: Did anyone think of any other expression?

Danilo: We did 2 times $(n-1) + 1 + n$.

Rô: Maybe that will also work... What does "n" mean?

Leonardo: The number of peaks.

Rô: Let's see. In the first figure, do 2 times $(2-1) + 1 + 2$. How do you resolve that?

Leonardo: First you do the operation in brackets, then the multiplication.

Rô: Oh, then it is $2 - 1 = 1$ and $2 \cdot 1 = 2$. And then?

Leonardo: $2 + 1 + 2 = 5$.

Rô: Let's check the second one too: $2 \cdot (3 - 1) + 1 + 3$. First, I'll do $3-1 = 2$, then $2 \cdot 2 = 4$ and $4 + 1 + 3 = 8$. How did you come to that expression?

Danilo: We tested $2 \cdot (n-1)$, but it didn't work. Then we thought of how it would have to be to work and we got that expression.

Rô: But can we reduce the expression?

Luana: We can reduce the "n" into one.

Rô: But how will we do that?

Leonardo: $2n$?

Rô: Is it? Do you think the expression can be reduced or not?

Leonardo: Yes, but we need to think.

Rô: Then let's think together. If I say that I went for groceries and got 2 pineapples, 1 watermelon, then I bought 2 pineapples and 3 watermelons, then 10 pineapples and 2 pears... Couldn't I have reduced all of that instead of saying pineapple and watermelon again and again?

Students: Yes.

Rô: How?

Leonardo: Putting it all together.

Rô: And can we put it all together in that expression?

Jucelena: You can do it like that (gesticulating with her hands on the air, indicating the distributive property of multiplication in relation to sum)

Rô: Doing it like that has a name.

Danilo: Distributive.

[...]

This moment of socialisation is part of a larger dynamics of interaction in the group, supported by the perspective of Van de Walle (2009) regarding moments in the classroom. One of these moments is when each student or group presents their own strategies and answers in the classroom and the teacher conducts this moment by asking questions so that the students talk about the task and reflect on the strategies created, seeking to systematise the ideas circulating there. In that dialogue, she challenges the students of two groups to communicate their ideas to the rest of the class and stimulates everyone to partake in the discussion. She asks them to verify if the laws they found are valid and tries to direct them towards the equivalences between these laws. She does not provide answers, but places questions that follow the students' manifestations, always with a question or stimulus to make them notice something new. In her communication with students, all of them are active participants: "from the beginning, the speaker [the teacher] waits for their answers [students'], expects an active responsive comprehension. It is as if

all the formulation was built from that answer” (Bakhtin, 2011, p. 301). We can thus see that students are active and attentive to her interventions.

The excerpt reveals a problematising teacher who invites her students to think with her while analysing their mathematical discourses in order to help them progress. In that context, the appropriation of the other’s discourse is indispensable for the re-signification of the teacher’s own discourse. That shows that we learn from one another, we need each other to constitute ourselves in an Alteritarian perspective of education. Sobral and Giacomelli (2020, p. 10) investigate alterity from the point of view of the dialogical relation between teacher and students: “It is a reflection founded on the idea that if we are altered by the other, we should move towards the other and let ourselves be penetrated by such alterity (Bakhtin, 1979), so that we return to ourselves enriched and having enriched our contacts”. A dialogical education, as these authors defend, is Alteritarian. In that perspective, there is a co-construction of knowledge: the teacher and students learn within interpersonal communication (Sfard, 2001). In co-construction, there are counterparts, as the value of each subject is recognised, each one contributes with his or her possibilities in a responsive way, building one another’s knowledge. Thus, the students’ mathematical discourse is intertwined with the teacher’s.

THE TEACHER’S PROFESSIONAL DEVELOPMENT

As mentioned, we endorse a Vygotskian conception of development that sees it as a social and interpersonal happening driving the internalisation of cultural forms of behaviour that affect intra-psychological activity. That social happening is nevertheless mediated by formulations that affect it and are affected by it. In this section, therefore, we will direct our perspective towards formulations that emerge along the pedagogic narrative, seeking to identify hints of professional development and of the instruments that mediated that process.

Through the pedagogic narrative, the teacher produced reflections connected to knowledges of experience (Contreras Domingo, 2013). We understand professional development as a form of knowledge that, although constructed in intersubjective relations and depending on them, is particular, subjective, personal, and tied to the singularity of each person’s experience.

Analysing the teacher’s narratives, we identify hints of knowledge from experience in two distinct moments. First, when she explains the reasons, she chose to begin the work with the development of algebraic thought with task 1: “*The first, simpler task was used to trigger a more complex investigative work of regularities during the second task*” (Frare, 2019, p. 162). Identifying that students were not used to that kind of proposal, the teacher began the work with a simpler proposal in order to bring them closer both to the type of task and to the observation and generalisation of regularities. Secondly, the teacher reports that when she noticed that the groups did not reach a law of formation that described the regularity, she decided to problematise: “*Instead of saying all that, wouldn’t it be simpler to find a mathematical expression? With that, this group sought to generalise the situation using a law of formation*” (Frare, 2019, p. 166). Noticing that the strategies created could be generalised, the teacher intentionally set a



problematization for students that aimed to mobilise a reflection and a reformulation that would then lead them towards generalisation.

Other instruments mediate the teacher's professional development, as the appropriation of theoretical discourses in Mathematical Education and practices related to the dynamics created in the group. For instance, she says: "*Like Hiebert et al. (1997), I see choosing the tasks as part of the teacher's role. They must constitute challenges for the students, allowing the exploration of mathematical thought and resolution strategies*" (Frare, 2019, p. 161). In that piece of speech, she demonstrates the appropriation of a discourse by Mathematical Education, but also of a group practice that consists in elaborating or adapting challenging tasks so they allow exploring and producing different strategies that foster the development of mathematical thought.

The appropriation of discourses by Mathematical Education appears again when the teacher talks about the tradition of Mathematics class based on the "paradigm of the exercise" and on the practice of stimulating writing as a form of registering and organising thought:

As I distributed the sheets, they wanted to know the "operation" they would be asked to do and what exactly they should write in the sheet. This is because Maths classrooms are traditionally based on the "exercise paradigm" (Alro & Skovsmose, 2006). (...) Most times, these exercises are done individually and corrected afterwards, and they present only one correct solution. There is no space for socialising the works in groups, problematising, investigating, problem-solving, etc.

In that perspective, there is also no place for writing as a form of registering resolution strategies, and only the result is meaningful, not the process. Nevertheless, it should be taken as "an important tool to develop and foment mathematical learning" (Powell & Bairral, 2006, p. 101), a connection that potentiates learning. (...). Thus, I had to encourage students to register their ideas and resolution strategies instead of only presenting the results of a series of calculations. (Frare, 2019, p. 163)

Pedagogic mediation, an exercise in which the teacher drives the students' perceptions towards questions that help advance their hypotheses by means of problematisations and questions, is a practice of the group and it appears in the teachers' practices when she states that:

Besides that incentive, I also did other things during the development of the tasks, such as mediating, problematising, and making students reflect. As I walked around the groups, I did not provide information or answers, but sought to instigate them to notice regularities, make generalisations, and perceive algebraic thought. In other words, I tried to make them investigate, raise hypotheses for solutions, and make them effective. (Frare, 2019, p. 163)

It is also worth stressing that it is indispensable that, during the process of mediation, the teacher asks good questions, helping students make progress. Problematisations make it possible for them to reflect on things they haven't previously thought about and seek

relations in order to answer the questions. Besides unleashing algebraic thought, such movement allowed the production of meaning and conceptual elaboration (Frare, 2019, p. 176). Students also influenced the teacher's process of professional development when, in the course of producing a hypothesis to the problem she constructed, they posed unpredicted questions, leading her to reflect on the task, identify its unexplored potentialities, and possibly reformulate it. Reformulating the task is also one of the practices produced within the group dynamics.

[...] I realised I could have suggested a question about the relations between the number of peaks in each wall, the number of cubes, and the number of the figure, given that some groups have found such a relation even if it was not proposed or intended. This is a potentiality of the task and I'll come back to it later. They also raised a variety of strategies for determining a term in a sequence of figures, recognised different laws of formation for the same sequence, and noticed the equivalence among them by means of my mediation. (Frare, 2019, p. 168)

I reflected on the situation and realised that besides the choice and adaptation of the tasks, they must be applied considering possible reformulations. Thus, I think that the items proposed in Task 2 one more could be added, seeking to establish the relation between the number of peaks in a wall, in the figure, and the number of cubes in the wall it represents, and, consequently, identify the respective law of formation. (Frare, 2019, p. 176)

Also related to the dynamics of group work is the valorisation of the different strategies employed by the students and looking for ways to explore them in order to mobilise a reflexive movement by means of socialisation. This practice appears in the teacher's discourse when she affirms that:

It is the teacher's role to make an environment of communication and reflection possible so that these strategies are known, verified, and discussed by the other groups, seeking to understand algebraic knowledges.

During the socialisation of the tasks, it was evident that different groups found various laws of formation for the same sequence. Then, I sought to mediate them into realising the equivalence among them. (Frare, 2019, p. 170)

Through the narrative, we identify several formulations that show the teacher's professional development. Due to the space limitations, we selected a few excerpts in which we see such discourses intertwined with the voices of the groups, inspiring pedagogic practices and mobilising a reflexive look upon the process in the classroom; the authors' voices, creating a theoretical lens that helped analyse experiences and their re-significations; the students' voices, who mobilised reflections, decision-making, reformulations, and re-significations within their hypotheses.



CONCLUSION

In this article, we analysed the narrative of a mathematics teacher who participates in a collaborative group as she worked with tasks related to algebraic thought with her high school students. We sought hints of how the collaboration that existed in the group becomes a practice of education and enables the professional development of the mathematics teacher involved in it.

The narrative of teacher Rosangela shows how she appropriated the principles defended by the group related to the social culture of a mathematics class (Hiebert et al., 1997), valuing the type of task she proposed, her role as mediator of their learning processes, and placing herself in a position to listen to them and value their mathematical discourses. As for the tasks she proposed, we can state that they have contributed to the mathematical agency of the students in Powell's conception (2019):

We define it as mathematical discursive and inscriptive ideas and forms of reasoning made clearer by the individual or collaborative initiative of the learners to define or redefine and build or go beyond the specificities of the mathematical situations in which they were invited to work. (pp. 193-194)

In the dialogues she establishes with her students, it is possible to identify the moments of co-construction of knowledge, given that she does not only problematise the formulations the students have about the task, but also presents herself as a learner, collaboratively analysing unforeseen strategies with them. Thus, in the cultural-historical perspective adopted in the text, the teacher demonstrates how an adequate intervention enables advances in students' learning and development.

It was also possible to identify her professional development and agency in several moments of her narrative (Oliveira & Cyrino, 2011).

We understand that participation in a collaborative group that constitutes a community of investigation (Jaworski, 2008) might enable the mediation of professional agency, depending on the selected methodology for this work. Teacher Rosangela's reports suggest how the identity of the group constitutes her identity and, as Oliveira and Cyrino (2011) declare, there is a strong connection between agency and professional identity. Practiced by the teacher, agency allows him or her to "make decisions that affect the performed work and reveal ethical and professional commitments, by means of ideas, motivations, interests, and goals" (Cyrino, 2017, p. 706). In her narrative, Rosangela reveals her political and ethical commitment with her students, building a communicational and discursive practice (Sfard, 2001) in a dialogic process of collaborative and reciprocal learning. Her narrative shows evidence of production of practice-related knowledge (Cochran-Smith & Lytle, 1999); a local form of knowledge that, when shared in a group, enables its own insertion into a broader field of mathematical teaching practices. The production of the narrative is a process of teacher education for the author and another teacher with whom she shares it.

Analyses of narratives allow us to affirm the potentiality of a collaborative group for promoting agency and the professional development of its participants, corroborating the ideas of Sancar et al. (2021). We understand that the group needs to employ an established work methodology with the intention of building a formative



environment. With its iterative cycles and by producing pedagogic narratives (Chapman, 2008) with its participants, the Design Research methodology has empowered them professionally. The present study meets the proposals of Haug and Mork (2021) that professional development takes time; we add that collaboration in a group happens throughout its own trajectory, with interactions that drive learning and promote professional development.

REFERENCES

- Abreu, A. et al. (2014). *Obmep: banco de questões- 2014*. Rio de Janeiro: Impa.
- Anderson, T., & Shattuck, J. (2012). Design-Based Research: A Decade of Progress in Education Research? *Educational Researcher*, 41(Jan/Feb.), 16-25. <https://doi.org/10.3102/0013189X11428813>
- Bakhtin, M. (2011). *Estética da criação verbal*. (6ª Edição). São Paulo: Martins Fontes.
- Bannan-Ritland, B. (2008). Teacher design research: An emerging paradigm for teachers' professional development. In A. E. Kelly, R- A. Lesh & J. Y. Baek (Eds.), *Handbook of Design Research Methods in Education* (pp. 246-262). New York: Routledge.
- Boavida, A. M., & Ponte, J. P. (2002). Investigação Colaborativa: potencialidades e problemas. In GTI (Org.), *Refletir e investigar sobre a prática profissional* (pp. 43-55). Lisboa: Associação de Professores de Matemática.
- Chapman, O. (2008). Narratives in mathematics teacher education. In D. Tirosh & T. Wood (Eds.), *The international handbook of mathematics teacher education: Tools and processes in mathematics teacher education* (Vol. 2, pp. 15-38). Dordrecht: Sense Publishers.
- Cochran-smith, M., & Lytle, S. L. (1999). Relationships of knowledge of practice: teacher learning in communities. *Review of Research in Education*, 24, 249-305. <https://doi.org/10.2307/1167272>
- Coelho, M. A. V. M. P. (2017). Grupos colaborativos na formação de professores: uma revisão sistemática de trabalhos brasileiros. *Zetetiké*, 25(2), 345-361. <https://doi.org/10.20396/zet.v25i2.8647600>
- Contreras Domingo, J. (2013). El saber de la experiencia en la formación inicial del profesorado. *Revista Interuniversitaria de Formación del Profesorado*, 27(3), 125-136.
- Cyrino, M. C. C. T. (2017). Identidade Profissional de (futuros) Professores que Ensinam Matemática. *Perspectivas da Educação Matemática*, 10(24), 699-712.
- Fiorentini, D. (2004). Pesquisar práticas colaborativas ou pesquisar colaborativamente? In M. C. Borba & J. L. Araújo (Orgs.), *Pesquisa Qualitativa em Educação Matemática* (pp. 47-76). Belo Horizonte: Autêntica.



- Frare, R. E. B. (2019). Tarefas sobre sequências no ensino médio: identificando regularidades e fazendo generalizações. In A. M. Nacarato & I. A. Custódio (Orgs.), *Narrativas de aulas de Matemática de uma comunidade de investigação como prática de formação docente* (pp. 161-177). Brasília: Sociedade Brasileira de Educação Matemática.
- Hargreaves, A. (1998). Os professores em tempos de mudança: o trabalho e a cultura dos professores na idade pós-moderna. Lisboa: MacGraw-Hill.
- Haug, B. S., & Mork, S. M. (2021). Taking 21st century skills from vision to classroom: What teacher highlight as supportive professional development in the light of new demands from educational reforms. *Teaching and Teacher Education*, 100, 1-12. <https://doi.org/10.1016/j.tate.2021.103286>
- Hiebert, J. et al. (1997). *Making sense: Teaching and learning mathematics*. Heinemann: Portsmouth.
- Ibiapina, I. M. L. M. (2008). Pesquisa colaborativa: investigação, formação e produção de conhecimentos. Brasília, DF: Liber Livro Editora.
- Jaworski, B. (2008). Building and sustaining inquiry communities in mathematics teaching development: teachers and didacticians in collaboration. In K. Krainer & T. Wood (Eds.), *The international handbook of mathematics teacher education* (Vol. 3, pp. 309-330). Rotterdam: Sense Publishers.
- Molina, M., Castro, E., & Castro, E. (2007). Teaching Experiments within Design Research. *The International Journal of Interdisciplinary Social Sciences: Annual Review*, 2(4), 435-440.
- Oliveira, H. M., & Cyrino, M. C. C. T. (2011). A formação inicial de professores de matemática em Portugal e no Brasil: narrativas de vulnerabilidade e agência. *Interações*, 18, 104-130. <https://doi.org/10.25755/int.461>
- Passos, C. L. B., Nacarato, A. M., Fiorentini, D., Miskulin, R. G. S., Grando, R. C., Gama, R. P., & Melo, M. V. (2006). Desenvolvimento profissional do professor que ensina Matemática: uma meta-análise de estudos brasileiros. *Quadrante*, 15(1-2), 193-219. <https://doi.org/10.48489/quadrante.22800>
- Powell, A. (2019). Tarefas para promover e dar suporte à Agência Matemática de Aprendizizes. In A. M. Nacarato & I. A. Custódio (Orgs.), *Narrativas de aulas de Matemática de uma comunidade de investigação como prática de formação docente* (pp. 190-208). Brasília: Sociedade Brasileira de Educação Matemática.
- Powell, A., & Ali, K. V. (2018). Design Research in mathematics Education: investigating a measuring approach to fraction sense. In J. F. Custódio et al. (Orgs.), *Programa de pós-graduação em educação científica e tecnológica (PPGECT): contribuições para pesquisa e ensino* (pp. 221-242). São Paulo: Editora Livraria da Física.
- Robutti, O., Cusi, A., Clark-Wilson, A., Jaworski, B., Chapman, O., Esteley, C., Goos, M., Isoda, M., & Joubert, M. (2016). ICME international survey on teachers working and learning through collaboration. *ZDM Mathematics Education*, 48, 651-690. <https://doi.org/10.1007/s11858-016-0797-5>

- Sancar, R., Atal, D., & Deryakulu, D. (2021). A new framework for teachers' professional development. *Teaching and Teacher Education*, 101, 1-12. <https://doi.org/10.1016/j.tate.2021.103305>
- Sfard, A. (2001). There is more to discourse than meets the ears: looking at thinking as communicating to learn more about mathematical learning. *Educational Studies in Mathematics*, 46, 13-57. <https://doi.org/10.1023/A:1014097416157>
- Smolka, A. L. B. (2000). O (im)próprio e o (im)pertinente na apropriação das práticas sociais. *Cadernos Cedes*, 50, 26-40. <https://doi.org/10.1590/S0101-32622000000100003>
- Sobral, A., & Giacomelli, K. (2020). Educação dialógica alteritária: uma reflexão. *Linguas e Letras*, 21(49), 7-21. Retirado de: <http://e-revista.unioeste.br/index.php/linguaseletras/article/view/24675>
- Van de Walle, J. (2009). Matemática no ensino fundamental. Formação de professores e aplicação em sala de aula. (6ª Edição). Porto Alegre: Artmed.
- Vygotski, L. S. (2007). *A formação social da mente*. (7ª Edição). São Paulo: Martins Fontes.

*

Received: December 14, 2020

Revisions required: April 2, 2021

Accepted: June 1, 2021

Published online: June 30, 2021

