ESSAY ON THE ROLE OF TEACHERS' QUESTIONING IN INQUIRY-BASED MATHEMATICS TEACHING

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ABSTRACT

This paper is an essay on the role of the mathematics teacher's questioning in inquiry-based teaching. Questions are important communication tools that are used by the teacher for various purposes and underpin different visions of what it means to teach mathematics. Inquiry-based mathematics teaching has achieved relevance as a powerful alternative to direct teaching, which is inefficient in complying with current demands of mathematics learning. The paper constitutes a reflection on teachers´ questioning within an inquiry-based approach to teaching mathematics, based on available research and illustrated by classroom episodes of three basic education teachers. Our reflection has led us to advocate the central role of the teacher's questions in inquiry-based mathematics teaching, having two main goals: (i) verification of knowledge, a questioning goal that is common to the direct teaching approach; and (ii) development of knowledge, a questioning goal that is specific to inquiry-based teaching. These two goals are attained using three types of questions which may be present in all phases of an inquiry-based lesson, albeit with different weights according to the lesson phases and the teacher's own goals.

KEY WORDS

Mathematics communication; Questioning; Teacher;
Inquiry-based mathematics teaching.

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INTRODUCTION

Questioning has long been a tradition in teacher discourse and has played an important role in structuring teaching activities (Gall, 1970; Menezes, 1996, 2004; Nicol, 1999; Tomás Ferreira, 2005). In the direct teaching approach, most of the teacher's questions are aimed at testing students' knowledge, and are usually posed after an initial presentation of content made by the teacher (Ainley, 1988; Mason, 2010). Inquiry-based mathematics teaching is characterized by a strongly interactive discourse and by new roles played by both teachers and students. Such an approach to teaching mathematics has gradually gained relevance (Canavarro, Oliveira & Menezes, 2012; ME, 2007; Ponte, 2005). However, there has not yet been sufficient examination of this approach to teaching mathematics, when it comes to teacher discourse and the teacher's use of questions. Thus, as part of the project entitled P3M – *Professional Practices of Mathematics Teachers* – which studies, among other issues, the communication practices of mathematics teachers, we aim to discuss the role of teacher questions in inquiry-based mathematics teaching.

In this essay, we start by laying the grounds on communication and inquiry-based mathematics teaching. We then advocate for the specificity and centrality of teacher questioning in an inquiry-based mathematics classroom, perceiving it as a discursive tool for learning.

COMMUNICATION AND INQUIRY-BASED MATHEMATICS TEACHING

Communication is a structuring element of human activity. In particular, it is a structuring element of the act of teaching. Since we do not live in isolation but rather in interaction with others, our activity is characterized by a strong communicative element. In other words, much of what we do is, or involves, communicating. Given that communication is closely related to our daily actions, the decisions that we make at every moment, which lead us to choose one path over another, are motivated by our vision of what surrounds us, in particular by our conception of communication (Brendefur & Frykholm, 2000).

Mathematics teaching is effected through a set of actions carried out by the teacher, supported and justified by the teacher's knowledge of mathematics, of students and their forms of learning, of curricula, and of instructional practice (Ponte, 2012). Such knowledge transversally embodies the idea of communication as a generative and as a disseminative element (Sierpinska, 1998). Communication is embedded in the generation and representation of mathematical knowledge. At the same time, communication plays a central role in the teaching and learning of mathematics.

In our paper, we discuss some of the main conceptions of mathematical communication in the classroom. Then we focus on the inquiry-based approach to teaching mathematics, which embodies one of those conceptions.

MATHEMATICAL COMMUNICATION IN THE CLASSROOM

When analysing the mathematical communication that occurs in the classroom – globally viewed as the communication that focuses on mathematical ideas and uses mathematical processes and representations – we can identify two main conceptions of mathematical communication. One sees communication as *transmission* of information, knowledge and ideas, a process that is anchored in knowledge and in the various forms of disseminating it. The other conception views communication as *social interaction*, in which the subjects interact with each other, searching shared meanings, and collectively constructing knowledge and forms of circulating it (e.g., Bauersfeld, 1994; Godino & Llinares, 2000; Sierpinska, 1998). The existence of communicative relationships amongst those who communicate (which occur in a certain context and involve the use of a shared code) is assumed by these two visions of communication, but they are distinguished essentially by the intentions of those who communicate. Thus, mathematics communication in the classroom assumes the existence of knowledge, culturally shared codes and relations among the actors (i.e., among the students and between these and the teacher). It is, thus, an essentially communicative process that can be either a transmission/circulation instrument for mathematical knowledge, employing its own language, or a basis for the social construction of mathematical knowledge amongst different actors in the classroom. Sierspinka (1998) clarifies this divergence: «From the interactionist perspective, transmission of knowledge is not an issue because knowledge is not in the head of the teacher. It is something that emerges from shared discursive practices that develop within the cultures of the classroom» (p. 57).

When communication is seen as a transmission (as a tool), its goal of communication is to persuade the other. Thus, based on a relationship of authority, the sender intends for the receiver to react as predicted, in accordance with the message sent. It is important that the message be preserved as much as possible, avoiding noise, in order to ensure that the receiver gets the message with the greatest possible accuracy in terms of the sender's intentions (Bitti & Zani, 1997). Under such a vision of communication, the interlocutors act neutrally toward what is being communicated, and the act is labelled «message decoding» instead of «interpretation». This perspective of communication entails the existence of a mathematical knowledge, previously coded by the teacher, transmissible to the students, in a culturally recognizable language, through constant noise reduction, regardless of how many students are in the classroom.

However, when communication is seen as social interaction (as foundation), it is a social process in which the subjects interact with each other, exchanging information, influencing one another, but looking to build shared meanings. This is a process of successive approximations, in which both parts supply additional information which helps to construct meaning through interpretation. In this perspective, mathematical knowledge emerges from collective processes of communication and interaction among the subjects and the classroom culture, including the teacher's interactions with the students about mathematics (Sierpinska, 1998). The social interactions amongst the students and between them and the teacher are fundamental in the mathematics teaching and learning process, namely in the interpretation and negotiation of social and mathematical meanings (Bauersfeld, 1994). The students' mathematical knowledge is influenced by the nature of the communicative actions happening in the classroom and is, therefore, socially constructed and conditioned by the teacher's and the students' ability to understand, reflect, negotiate meanings, and establish mathematical connections.

These two perspectives of communication match general orientations for teaching practices. In daily classroom life, we can find evidence of each of these perspectives, but also intermediate forms of communication (Brendefur & Frykholm, 2000).

INQUIRY-BASED MATHEMATICS TEACHING

There are several differences between inquiry-based and direct teaching. These two approaches to teaching mathematics are distinguished essentially by the mathematical communication generated in the classroom, the teacher's and students' roles in classroom activities, the status of mathematical knowledge, and the tasks that are posed to students and developed by the class as a collective group (Ponte, 2005). In inquiry-based teaching, «the emphasis is moved from the 'teaching' activity to the more complex activity of 'teaching and learning'» (Ponte, 2005, p. 13). The teacher's role is no longer merely to transmit mathematical knowledge to attentive and silent students. Above all, the teacher is expected to engage the students in rich mathematical activities based on challenging mathematical tasks, working autonomously (usually in small groups) and also collectively (with the whole class), emphasizing discussion and negotiation of meanings (Bishop & Goffree, 1986; Ponte, 2005).

An inquiry-based mathematics lesson is usually organized around three or four phases, according to whether or not the last phase is unfolded. Stein, Engle, Smith, and Hughes (2008) propose a three-phase model (the *launch* phase, the *explore* phase, and the *discuss and summarize* phase), while Canavarro *et al.* (2012) advocate four phases, emphasizing the systematization of mathematical learning as a phase of particular importance. In each of these phases, the teacher carries out a set of actions directly aimed at promoting mathematical learning and a set of actions targeting classroom management. The actions aimed at fostering mathematical learning have a greater impact on the classroom discourse and mathematical communication. Next we describe each of those four phases.

In the first phase of the lesson – *Introduction of the task* – the teacher looks to ensure that the students appropriate the task (by acquainting them with the context and interpreting the task, namely its goals, while avoiding reducing the task's cognitive demand) and to promote task engagement. As far as management is concerned, the teacher organizes the students' work (individually, in pairs, in small groups), creating an adequate environment for students to complete the task (for example, supplying materials that support students' work).

In the second phase – *Development of the task* – students work autonomously, usually in small groups. In this phase, the teacher must guarantee that the students complete the task by posing questions, offering hints, suggesting forms of representations, and asking for clarification and justification. The teacher must also maintain the cognitive demand of the task and stimulate students' autonomy by fostering mathematical reasoning and avoiding validating answers. In terms of management, the teacher promotes pair and group work, regulating students' interactions and asking them to keep a record of all their work to support the collective discussion.

The *Discussion of the task* is a rather important phase in an inquiry-based lesson that goes beyond the presentation of solutions. It is a rich moment concerning mathematical communication and the search for common grounds, whose ultimate aim is the construction of knowledge. Canavarro *et al.* (2012) emphasize, in this phase, the teacher's intention of promoting the mathematical quality of students' presentations and regulating their interactions through questioning, asking for explanations and the underlying rationale behind the strategies and reasoning presented. As far as classroom management is concerned, it is crucial that the teacher maintains an environment conducive to the discussion of ideas by managing students' participation and encouraging the sharing of mathematical ideas, regardless of whether they are incomplete, confusing or even wrong. The aim is thus to deconstruct incorrect knowledge and construct mathematical knowledge in a precise language that is recognizable to the students.

The main mathematical ideas that are discussed and shared in the previous phase are expected to be recalled, systematized, and recorded during the last phase – *Systematization of mathematical learning*. In this phase, the teacher, with the students' collaboration, institutionalizes ideas or procedures and establishes connections with the students' own knowledge. This is done by means of actions such as identifying representations and pointing at connections to previously learned concepts. In terms of classroom management, the teacher must focus students' attention on the systematizing activity and ensure that the ideas emerging from that activity are recorded in written form. It is important to note that this phase of systematization of mathematical learning does not necessarily occur after the discussion of the task. In some cases, as the lesson unfolds, the systematization of mathematical ideas may be simultaneous with the discussion of the task. In addition, there may be several moments of discussion/systematization during the development of the task. For example, if there is a generalized question or mistake, the teacher may stop the students' activity in order to discuss the issue before resuming the task.

Inquiry-based mathematics teaching is underpinned by a conception of communication as social interaction. Thus, it presents the teacher with a set of challenges with regard to the management of his own discourse and that of the students. Questioning is an important element of the teacher's discourse (and also a challenge), and this is what we address next.

QUESTIONING: A FACET OF THE TEACHER'S DISCOURSE

Discourse can be seen as language in action, that is, the usage of a linguistic system in real contexts with the goal of communicating (Sierpinska, 1998). According to Searle (1984), «speaking a language is performing acts according to rules, acts, acts such as making statements, giving commands, asking questions» (p. 26). The teacher is always a producer of discourse in the mathematics classroom. This discourse, which is substantiated through different communicative actions, may be of distinct nature according to the teacher's perspectives on mathematics teaching and learning and, in particular, on the role played by communication in these processes. Thus, «the discourse of the mathematics class reflects messages about what it means to know mathematics, what makes something true or reasonable, and what doing mathematics entails» (NCTM, 1991, p. 54). The teacher's communicative actions in a mathematics classroom may be quite varied: questioning, explaining, listening, responding (Nicol, 1999; Tomás Ferreira, 2005). In this paper, though recognizing the strong interrelationship amongst these actions, we focus on questioning, since it is a powerful promoter of student discourse.

We start by discussing the concept of questioning and its related terminology. Do the different terms *question*, *interrogation*, *query*, *demand*, *inquiry*, represent the same thing? As far as teaching practice is concerned, but also, to a great extent, in the field of mathematics education research, these terms are used interchangeably, describing an action by which one person asks information of another. Pereira (1991) sees the question as being an *interpellation*, which she defines as a «non-assertive enunciation – at least in its most common form – which corresponds, in some way, to the solicitation of a particular student or set of students who form a class» (p. 168). According to this author, interpellations may be questions demanding an answer, but they can also be orders or requests («Would you mind to step aside, so that your partner can look at the board?»), or an oral expression aimed at holding the students' attention («Ok?», «Isn't it?», «Right?»). Therefore, on the one hand, we have interpellations which, though formally interrogative, are not really questions since a verbal answer is not expected. On the other hand, we have interrogative enunciations, which we consider questions but which are, indeed, requests for information («Tell me what you are thinking») (Mason, 2010). In this paper, we consider all enunciations, interrogative or not, which reflect an actual request for information as questions. Thus, they are followed by a waiting time so that the answer may emerge.

Questioning has a strong presence in the practices of mathematics teachers. Yet, the teaching practices in general, and questioning practices in particular, of mathematics teachers with different perspectives of teaching and learning are, themselves, distinct. This distinction lies essentially on the purposes with which teachers ask questions of their students as well as on the moments in which that questioning occurs.

The roles played by students and teachers in direct or inquiry-based teaching are essentially different. In a mathematics classroom where the direct teaching approach has been adopted, all mathematical activities going on in the classroom are somehow focused on the teacher. Students, on the other hand, are supposed to listen to the teacher's explanations and reproduce his mathematical procedures. With this approach, questioning is an activity reserved for the teacher only and, in general, aims to test the students' knowledge.

On the contrary, in an inquiry-based mathematics classroom, it is the teacher's responsibility to propose learning situations that will help students to build their own knowledge. This is achieved not only by developing different actions aimed at promoting student learning but by placing the centre of mathematical activities in the hands of the students as a collective. Teacher and students question and listen attentively to each other, within a classroom culture that emphasizes sharing strategies and negotiating meanings. Therefore, questioning is an activity shared by all the classroom actors – teacher and students – who have different aims that go beyond testing the students' knowledge and scholastic achievement.

Hence, the relevance of questioning to the teacher's role in an inquiry-based approach to mathematics teaching is not surprising (Cengiz, Kline & Grant, 2011; Hufferd-Ackles, Fuson & Sherin, 2004). In particular, the teacher's questions challenge students to become active in the classroom through verbalization (presenting information orally or in written form) and reflection (analysing and weighing available information). As such, the teacher's questions may aim at either to verify the student's knowledge, or focus the students' attention on mathematical ideas or strategies, or even inquire them about how they are thinking. With these different purposes in mind, we can pinpoint three main types of questions: *verification*, *focusing*, and *inquiry* questions (Ainley, 1988; Mason, 1998, 2000).

Verification questions aim to test students' knowledge (which explains why they are also called *testing* questions), leading to short and immediate answers. These answers are previously idealised by the teacher, who believes that such questions contribute to regulate the way students learn mathematics. The teacher (the adult) builds a mental representation of the student's (the child) knowledge through verification questions, which test the knowledge supposedly acquired in the mathematics classroom. Verification questions also contribute to asserting the teacher's social control (Mason, 2010), namely when the teacher intends to regulate students' attitudes and behaviours in the classroom. However, when these questions have the latter aim, some authors refer to them as *pseudo-questions* (e.g., Ainley, 1988). In fact, with this type of query, no reply is expected, but rather some sort of enforced complicity (Mason, 1998).

Verification questions are quite common in the mathematics classroom. They play an important role in ascertaining knowledge acquisition, attesting to correctness, and articulating or interconnecting different ideas (Mason, 1998, 2000). This type of question may also include incomplete statements, made by the teacher, usually at the end of sentences, aiming to allow students to demonstrate what they know by completing the sentence (Menezes, 2004). This technique can promote the development of mathematical reasoning (by making sense out of the incomplete statement); but it can also result in mechanised routines (as with, for example, the recitation of multiplication tables).

Focusing questions aim to focus students' attention on a specific issue the teacher wants to underline. They can also be aimed to redirect the focus of attention to students' own reasoning. Such questions are specific to the educational arena and exhibit a strong formative intentionality. Hence, focusing questions do not usually appear in everyday life, as they are not genuine requests for information. In an educational context, however, focusing questions are a fundamental discursive tool that guides and supports student thinking, since they entail the effect of focusing or directing the attention of the audience (Mason, 2000, 2010). This sort of questioning may have a funnelling effect when students to not provide answers (Bauersfeld, 1994; Voigt, 1985), as they often generate a spiral of decreasingly difficult questions that only require of students short and quick answers. In the limit, this may be pushed to an unsatisfactory level with respect to the learning of mathematics, either when the teacher favours student involvement over fostering mathematical knowledge, or when the teacher maintains an excessively sophisticated level of mathematical discourse even if it is inaccessible to students. Focusing questions may assume a metacognitive dimension whenever they direct students' attention to their own thinking or when they deviate students' attention from the specificities of a task towards generalizing their mathematical ideas (Hufferd-Ackles *et al*., 2004; Mason, 1998, 2000, 2010). In such cases, focusing questions promote the students' mathematical knowledge in a broad sense.

Inquiry questions are in fact the genuine questions a teacher asks of students when seeking information. This type of question is used in everyday life to obtain informative content, except in situations of a social nature (e.g., «How are you?», «How have you been?») where the answers may already be known and, therefore, have no informative value (they are likely to be questions of circumstance), despite their relevance as a communication tool (Tropea, 2007).

For the teacher, «it is difficult to enquire genuinely about the answer to problems or tasks which have well known answers» (Mason, 2000, p. 15). Actually, genuine inquiry aims essentially at accessing students' thinking, understanding their strategy use, and challenging them to build new mathematical knowledge. Sometimes an inquiry question may signal students that something is wrong with their performance, in which case the real motive of inquiry is distorted (Mason, 2010). However, when inquiry questions are absent from a mathematics classroom, it is likely that the teacher is omniscient and that the whole class has a non-questioning attitude. In such classrooms, students are unable to build their own mathematical knowledge through analysis, conjectures, and justification of properties and generalizations (Mason, 1998, 2000).

Looking globally at these three types of teacher questions, we realize that verification and focusing questions are closely linked to the didactical process, and are less common in everyday life, outside the school context. Indeed, such questions, especially verification questions, when used in daily interactions among adults (this is usually not the case when a child interacts with an adult), often lead to communication problems due to discomfort that can be created among people. Inquiry questions are most frequent in everyday life but their present in the mathematics classroom varies (Ainley, 1988). In fact, inquiry questions are common when the classroom culture emphasizes problem solving, reasoning and communication; such skills are regarded both as valuable tools for learning mathematics and as curricular goals.

Analysing the purposes of each of the three types of questions we have discussed in terms of their relationship to students' mathematical knowledge, we notice that verification questions are retrospective in nature because they target students' pre-existing knowledge. In contrast, focusing and inquiry questions are forward-reaching in that they focus on students' developing knowledge, with the support of the teacher and the collective classroom.

TEACHER'S QUESTIONING IN AN INQUIRY-BASED MATHEMATICS LESSON

As was previously discussed, the inquiry-based mathematics lesson may be seen as divided into four phases, distinguishing the experiences of collective discussion from systematization of mathematical learning. As the aim of this essay is to discuss the role of teacher questions in inquiry-based mathematics teaching, the subject becomes clearer if the four phases are analysed separately. Thus, we address the following phases: (i) introduction of the task; (ii) development of the task; (iii) discussion of the task; and (iv) systematization of mathematical learning. The teacher has an important presence in the discourse of all of these phases, namely through the questions he or she poses. The different types of questions we have addressed in this paper occur at different times and perform different aims throughout the lesson.

Next, we discuss the teacher's questioning in each of the phases of inquiry-based teaching. We use examples and classroom episodes from three different classes[1] to illustrate our claims and ideas. The classification of questions in the following episodes is the one that an independent, external observer can infer taking into account both the structure of the question and the associated contextual information. All episodes have been videotaped, so that the viewer can experience the verbal interactions, gestures, intonation, and the facial expressions of all the interlocutors.

TEACHERS' QUESTIONS IN THE INTRODUCTION OF THE TASK PHASE

When introducing the task, the teacher looks for familiarizing students with the task itself, while possibly referring them to resources they may use and explaining what is expected of them. Therefore, in this initial phase, the teacher may feel the need to pose a number of questions to check whether the students have understood what is being proposed and are ready to start working autonomously (Mason, 2000; NCTM, 1991).

Through *verification* questions, the teacher can *ascertain students' prior knowledge* that is necessary to accomplish the task. The teacher may ask several verification questions, namely questions *centred on concepts* that are explicitly present in the task, or related concepts; questions pertaining to the task's *context*, eliciting students' experiences; questions to assess the understanding of the task's *goals and language*, be they mathematical (notations and terminology) or natural (Ainley, 1988; Nicol, 1999).

In order to ensure that the understanding of the task is not an obstacle to its accomplishment, teachers often feel the need to ask questions like: «Is there any word or expression you do not know?» Canavarro *et al.* (2012) refer

1 The three lessons discussed in this paper are part of the multimedia cases developed in the context of task 3 of the research project P3M – Professional Practices of Mathematics Teachers. They exhibit the characteristics of inquiry-based mathematics teaching (Canavarro, 2011; Ponte, 2005), in the sense that they are built upon students' activities, triggered by tasks posed by the teacher towards the construction of mathematical knowledge in a process emphasizing the discussion of mathematical ideas. The three teachers involved have a long professional experience (15 to 20 years), and usually enact inquiry based teaching.
The 1st cycle (4th grade) lesson, conducted by Célia, develops around the task «Cubes with stickers», and aims at developing algebraic reasoning. In particular, it focuses on the recognition of a sequence and its variables, the identification of their relationship, and the development of a corresponding general expression, expressing it both in natural and mathematical language. The 2nd cycle (5th grade) lesson, conducted by Fernanda, aims to deepen the students' understanding of the concept of percentages and its connection to the notion of unit (in the context of the Rational Numbers theme). It is based on the task «The rise and fall of fuel prices». Finally, the task «The class president´s election» is proposed by Cláudia to a 7th grade class in order to enrich students' knowledge of first degree equations. The three tasks can be found in the appendix.

to these questions as «how?»-questions because they intend to «clarify the way the task is interpreted» (p. 9).

In the 7th grade mathematics lesson, students were expected to identify who had won the election for president in a class with 30 students. When, after reading the task aloud, the teacher asks: «Any doubts?», followed by «Do you know what blank or null votes are?» (episode 1), she is precisely verifying the students' prior knowledge.

EPISODE I

Teacher: Let's read the task «The class president´s election»: (…) Any doubts? Do you know what blank and null votes are?

Students: Yes.

Teacher: What do you mean by that?

Students: They didn't count…

Student: They didn't vote.

Teacher: It's not that they didn't vote. Voting blank means that they have not chosen any candidate. A null vote is when a vote is erased or somehow damaged (…) Now, you can use different strategies to solve the task; don't forget you have to show how you worked it out. You have 10 minutes to finish the task.

By asking «Any doubts?», the teacher aims to pinpoint students' difficulties in understanding the task. This question, which is equivalent to «Did you understand?», sometimes does not produce the expected outcome, thus missing its purpose (Menezes, 1996). Often, students simply do not answer because they are unable to process all the information or are unaware of possible difficulties. Thus, the goal of the teacher's verification questions is better achieved by testing some specific aspects of the task, which the teacher anticipates as potential sources of misunderstanding (Ainley, 1988; Cengiz *et al.*, 2011). This is exactly what the teacher did when, after asking «Any doubts?», she added the verification question «Do you know what blank and null votes are?».

When the teacher addresses blank and null votes, her intention is to verify whether students have assigned a meaning to the task statement «All 30 students in the class have voted and no blank or null votes were cast». Knowing that there are no blank or null votes is, in fact, an important piece of information, although it may seem, at first, unnecessary. However, without a grasp of this fact, the problem could have multiple solutions, instead of just one (as intended). Therefore, the teacher found it relevant to make sure that this knowledge was shared amongst the students. Similarly, the question «What do you mean by that?» reinforces the previous query while challenging students to justify their answer. Since they are verification questions, the answers tend to be quite brief, as can be seen in episode 1 (NCTM, 1991; Tomás Ferreira, 2005; Wood, 1995).

The teacher may follow different strategies in order to check whether students are able to understand the task, the main purpose of this phase of the lesson. For example, she may ask them to present the task in their own words, or to check if any concept is unfamiliar to them (Canavarro, 2011). Episode 2, drawn from a 4^{th} grade lesson, illustrates the former strategy. The task «Cubes with stickers» aims to determine the number of stickers required to fill in the faces of rows of cubes, united by their faces. The teacher asks students to read the problem and explain it in their own words. In this way, she verifies if the problem is correctly understood, helps other students to clarify their own understanding of the problem by drawing on the interpretations of their classmates, and promotes the development of communication skills.

EPISODE 2

Teacher: Who is able to explain, in his own words, what this task is all about? João…

João: She is making constructions with the cubes and putting stickers in each visible face; but she didn't put any stickers in the middle; and they say there that she used 10 stickers and this is right because she didn't put any sticker in the middle of the cubes. They were together, so she can't… that is, she can but it wouldn't make much sense.

Teacher: Why wouldn't it make much sense?

Students: Because it couldn't be seen.

Teacher: So, in that construction, the one she made with 2 cubes, she used 10 stickers. I have one cube here, two cubes. Very quickly, let's see, how she would make this construction. Rita, do want to come here and help? How would Joana, we have a Joana here, make this construction? I have 2 cubes here, and glue …

In this episode, the teacher reinforces the initial proposal – «to explain in (…) own words» – with a direct question to the student: «Why wouldn't it make much sense?» This reinforcement is made even stronger since the teacher has decided to make the construction mentioned in the task statement herself by gluing the stickers with the help of a student.

Whenever tasks involve references to a real context, the teacher must also make sure the context is familiar to the students. For example, in the task dealing with the rise and fall of fuel prices (5th grade lesson), the teacher tries to draw the students' attention to the context by asking «Do you often put fuel in your parents' car?»

TEACHERS' QUESTIONS IN THE DEVELOPMENT OF THE TASK PHASE

The students work on their own to complete the task usually in pairs or small groups, since peer interactions and mutual support suit the cognitive level of challenging tasks. While monitoring students' activity, the teacher seeks to understand how they think, what meanings they ascribe to the mathematical ideas they are working with and the representations they are using, what justifications they present, and what difficulties they reveal in completing the task, etc. (Canavarro *et al.*, 2012). Thus, in this phase of the lesson, the purposes of the teacher's questions may include *verification*, *focalization*, and *inquiry*. Yet, it is likely that focusing and inquiry questions are predominant (Mason, 2000; Nicol, 1999).

One of the purposes of *inquiry* questions is to *access students' thinking and understand it*. Yet, we must bear in mind that these questions help to build a «*conjecturing* atmosphere» (Mason, 2010, p. 6), since the teacher deals with students' assertions or answers as if they were conjectures, not considering them right or wrong at the outset. One means of understanding students' thinking is to ask them to explain their ideas. In the following episode (episode 3), the 4th grade teacher asks Rita: «Why are you saying that?» In attempting to understand how Rita thought, she explicitly asks the student to explain her reasoning not only to herself but also to Diogo, who was working with Rita on the task. It is important to notice that the teacher also asks Diogo to pay attention to his classmate's explanation. In fact, as Rita verbalizes her thinking, the teacher's *hows* and *whys* emphasize the need for explanation and justification, while involving Diogo in the analysis of Rita's strategy.

EPISODE 3

Teacher: Why are you saying that? Explain it better, so that Diogo and I can understand it…

Rita: We took this one out and then we placed it here to make four cubes. Then we did like this: 4, 8, 12, 16, 17, and 18. 4 times 5 minus 2.

Teacher: 4 times 5. Why 5?

Rita: Because 1, 2, 3, 4, 5…

Diogo: Right, but then you're adding 2 cubes. So, you are saying that there are 2 cubes, one here, and another here.

Teacher: How do you think it should be?

Diogo: It should always be one less…

The following episode (episode 4), from a 7th grade lesson, involves the teacher posing inquiry questions. As in the previous episode, the ultimate purpose of these questions is to *understand how students think* while solving the task, which Nicol (1999) referred to as «posing questions to learn what students are thinking» (p. 53), or as Bishop and Goffree (1986) asserted: «the teacher's questions because he genuinely wants to know» (p. 329). We should realize that in episode 3 this goal is difficult for the teacher to achieve since students often fail to record the way they think or record it incompletely. The teacher tries to follow a group's strategy. Even though she recognizes a student's faulty reasoning, she asks her to explain the strategy that was followed by the whole group, insisting: «Okay… and so what?» As the student explains again her thinking, she realizes the flaw in her reasoning: «Ah, I know what was wrong!» Bishop and Goffree (1986) point out the explanatory value of this form of questioning: «This use of questioning by the teacher shows us that what is important about explaining is (…) that the *connections* get exposed – not that it is the teacher who necessarily does the exposing» (p. 334). In the end, the teacher realizes that, as she suspected, the students have followed a trial-and-error strategy.

EPISODE 4

Teacher: But you are not presenting your reasoning here! Okay, so you begin with 10, 10, 10. Is that it? Ten votes each. And then what?

Student: Then we know that Lucas got 2 votes less than Francisca, so we subtracted 2 from Lucas and added 2 to Francisca.

Teacher: Okay…

Student: Francisca gets 12 votes…

Teacher: Okay… and then what?

Student: Then Sandra got twice as many votes as Lucas, so what we did with Lucas… Ah, I know what was wrong! Okay… we need twice as many votes as Lucas'… twice as many votes as Lucas times 2… That was Sandra's number, Sandra's votes.

Teacher: So you're using trial and error, right? But show all the hypotheses. But you are thinking correctly.

Besides accessing students' thinking, inquiry questions also have another purpose, which entails *challenge* (Martinho & Ponte, 2009): to help students go further. For example, the 5th grade teacher stimulates students' thinking when she asks them: «Is it always going to be like that? Even with other prices? Affordable prices? Do you think so? Will it always happen like that?»

Focusing questions may cover several aspects. Some of them are *directly related to the task*, others to the *group dynamics* while students solve the task (Guerreiro, 2011). As for the aspects related to the task requiring the teacher's intervention, we find those dealing with difficulties in grasping the data of the problem, finding a solution process, or using language or representations. For example, the teacher may feel the need to focus students' attention on the task's wording. Without an accurate grasp of the specific statements used they may be unable to consider all the conditions needed to solve the problem and thus, hit a dead end (Menezes, 2004). In this sense (focusing on the data and problem conditions), the 7th grade teacher asks focusing questions of her students, such as «How do we use this 30?» and «What is this information for?», highlighting specific data in the mathematical problem students were working on.

Focusing questions naturally put emphasis on students' *mistakes* when solving mathematical tasks (Mason, 2000). In inquiry-based teaching, the goal of the teacher's questions is not to correct mistakes, but rather to help students identify and correct them by themselves (Wood, 1995). Having this in mind, Ms. Cláudia (7th grade teacher) goes through the students' notes to analyse their solution processes. As she identifies a processing error (such as $2 \times (-2) = 4$), she focuses the students' attention on the wrong answer to the multiplication, without immediately revealing the right answer, using a focusing question centred on the error «2 times − 2 … is equal to 4?» The students analyse and correct the product «No, it is −4».

In episode 5, 5th grade students working in groups believed they were thinking correctly: it was clear to them that if the price of fuel had risen by 10% and then fallen by another 10%, it would return to the original value.

EPISODE 5

Teacher: Tell me what you've done so far.

Student 1: (...) plus 10%, it will return to the previous price.

Teacher: Really? Have you tried it out?

Student: No.

Teacher: Yes or no?

Student 1: If the fuel is at a given price, then goes up by 10%... and then it goes down by 10%...

Student 2: It reaches a certain price, then the price is taken, it goes back to the same...

Teacher: But, have you checked that using a particular price? So, you think that if the price goes up by 10% and then falls by 10%, it will go back to its original value, is that it? Is everybody in the group thinking in the same way? And have you tried that with a specific price?

Student 2: We have to do it now...

The teacher pinpoints this error in the students' reasoning and asks them to focus their attention on the process they were using. She challenges them to check their conjecture with a specific price per litre. Although they acknowledge they had not worked it out using any specific price, it took them a while to be convinced that trying out a few prices would be a good idea. The teacher re(emphasizes) the focusing question when suggesting the students to experiment with a few specific prices. Yet, before posing the question, she made sure no one in the group disagreed with the initial (erroneous) conjecture.

In the two previous episodes (4 and 5), the teacher focuses the students' attention on the shortcomings of their reasoning. There are other situations in which the teacher questions the language (terms and notation) used by the students, both orally and in written form. In the 7th grade classroom, the teacher focuses students' attention on the inadequate use of certain terms. She revoices, in an interrogative way, a student's statement that incorrectly uses the word «annul» (regarding the parentheses) – «Do we have to annul?», concerning the discarding of parentheses in the simplification of an algebraic expression.

In episode 6, the teacher focuses her 4th graders' attention on the inadequacy of their *written mathematical language*.

EPISODE 6

Fábio: Yes, and then we did 4 times 5 equals 20, minus 2.

Teacher: Pay attention, 4 times 5 is equal to?

Marco: 20.

Teacher: So, is it equal to 20 minus 2? Does 4 times 5 equal 20 minus 2? Can you leave it like this?

All: No.

Teacher: No. 4 times 5 equals 20; 3 times 5 equals 15, right? But it is not 20 minus 2, I mean, 4 times 5. You have to separate them, don't you? How can you do that?

Fábio: By putting it in brackets…

Teacher: In brackets? 4 times 5 equals 20… then, what do you want to do?

Fábio: 20 minus 2.

Teacher: So, write that down here, below: 20 minus 2, which equals…

Fábio: … 18.

Teacher: 18, isn't it? You cannot write everything in the same line.

While students work cooperatively, the teacher realizes that they are writing down the process incorrectly. The students did not write correct numerical expressions for each of the actions made (4‘5 = 20, 20 - 2 = 18); instead, they wrote one single expression reflecting the set of those actions in sequence (4‘5 = 20 - 2 = 18), thus creating an incorrect and meaningless numerical expression.

The teacher's questions can also be viewed in the light of their two main goals in an inquiry-based lesson: to *promote learning* and to *manage interactions* (Menezes, Canavarro & Oliveira, 2012). In episode 7, the 7th grade teacher notices that the two elements of a pair are not really working together on the task (the election of the class president). Each of the students reveals different difficulties and makes different mistakes, though they each hold ideas that complement the other's. These ideas potentially help both students to correct each other and to find a solution to the problem together, provided they work cooperatively. The questions the teacher asks of them aim to not only focus the students' attention on their own mistakes and encourage them to identify and overcome their errors, but also help them to manage their joint interactions, while showing them that interacting has the potential to improve their performance.

EPISODE 7

Teacher: Do you agree, Beatriz? That the total of votes, plus Lucas's, plus Sandra's… this is what you wrote here…

Pedro: Plus Sandra's votes.

Teacher: Is that equal to Francisca's votes?

Beatriz: Wait a minute, I didn't get it…

Teacher: What he wrote here is that the total of votes, plus Lucas's … and there's something missing here, in the middle, plus Sandra's equals Francisca's. Do you agree with this equation?

Beatriz: No, I don't think so.

Teacher: So, how do you think you could write this in an equation?

As previously mentioned, in the development of the task phase, there is a prevalence of inquiry and focusing questions. Verification questions serve as supports for other types of questions, and sometimes help to resolve certain deadlocks. In this phase of an inquiry-based lesson, the role of verification questions is not so much one of testing or verifying, but rather one of supporting interactions of inquiry or focusing nature (Guerreiro, 2011). Thus, at a macro and more holistic level, the teacher's discourse in the development of the task phase is characterized mainly by inquiry and focusing questions; but it is also marked by some verification questions, especially when we look at the teacher-student interactions at a micro level.

TEACHER'S QUESTIONS DURING THE DISCUSSION OF THE TASK PHASE

In an inquiry-based mathematics lesson, the discussion of students' productions achieved during autonomous work, and the strategies and ideas employed requires the teacher to manage the students' discourse allowing everyone, including herself, to understand what is shared among the whole group (Cengiz *et al.*, 2011; Ruthven, Hofmann & Mercer, 2011; Stein *et al.*, 2008). The question is of paramount importance in attaining such goals as it serves to regulate discourse, leading the students to present information that the others do not know, which is one of the purposes of inquiry questions (Mason, 2000, 2010; Nicol, 1999). *Inquiry* questions are associated with *requests for explanation or justification* (Stein *et al.*, 2008; Yackel & Cobb, 1996) in the discussion of the task phase of an inquiry-based lesson. In this sense, the teacher challenges a group of 4^{th} graders to explain how they managed to find the pattern involved in placing a number of stickers on the cubes. She asks the following inquiry question: «What about for any given number of cubes? How would you find the number of stickers?»

Inquiry questions that elicit justification for ideas or procedures, allowing the teacher to learn about students' reasoning, are very common in inquiry-based mathematics teaching (Mason, 2010). Typically, these questions start with a «Why», and follow the students' own statements. In the 4th grade class (episode 8), the teacher repeatedly asks this type of question, seeking to gather information that will enable her to understand the students' thinking.

EPISODE 8

Caleça: If you remove plus 2, this is the 4 times table.

Teacher: Why is it always plus 4?

Caleça: Because you always do 4 times…

Teacher: But why?

Carolina: 9 times 4 equals 36. Then plus 2 makes 38. 10 times 4, 40; you add 2, 42. 2 is the number causing this…

Teacher: Number 2 is causing this. But why did you say … You have little arrows there, plus 4. But why plus 4 and not plus something else?

Carolina: Because the difference of 4…

Teacher: Why?

Focusing questions occur during the discussion of the task, usually when, during the explanation and justification of their ideas, the students display errors, imprecisions or lack of clarity (Guerreiro, 2011). In such situations, the teacher chooses to question the students, rather than point directly to the mistakes. Her intention is to have all the students re-examining their discourse (Nicol, 1999; Tomás Ferreira, 2005), acting in a similar way to that during the development of the task phase. In episode 9, the 5th grade teacher asks questions to clarify a student's explanation («Which, in your opinion, was how much?»; «Right, is it because otherwise 10% would still be 45 cents?») or to lead the student to conclude that the value found was incorrect («But, Rute, but the 45 cents that you are taking away from it, what is that?»).

EPISODE 9

Teacher: Rute, would you mind clarifying what you've just explained a bit more? Explain it better to us.

Rute: This A stands for the amount…

Teacher: Which, in your opinion, was how much?

Rute: It was € 4,50.

Teacher: Right, is it because otherwise 10% would still be 45 cents?

Rute: No.

Teacher: Bear in mind that you're adding 45 cents here. Okay, explain that to us.

Rute: Then, the previous amount of € 4,50 plus the 45 cents, which was 10% of that amount, would give us the price, a certain price, which is how much the fuel would cost including the increase. Then, that price together with the increase minus 45 cents would fall back to the same amount…it would lead to the same value…

Teacher: But, Rute, but the 45 cents that you are taking away from it, what is that?

Rute: Right, we made a mistake here because the 45 cents is related to € 4,50…

Verification questions have little weight during the discussion of the task phase. They usually occur when the teacher wishes to test the students' understanding of what has been presented, and often lay the grounds for the systematization of mathematical learning.

Teacher's questions during the systematization of mathematical learning phase

In this phase, the teacher combines the synthesis of the task's solutions, highlighting the appropriate usage of mathematical language (terms and notation), with possible extensions of the results obtained, often having in mind their mathematical generalization (Canavarro *et al.*, 2012). It is the moment at which mathematics learning becomes *institutionalized*, going beyond the task that has just been accomplished and attempting to systematize and to represent mathematical knowledge. At this phase of an inquiry-based lesson, which is not as rich in teacher questions as the previous phases, the teacher uses *verification* questions whose answers may indicate how well the students have understood the concepts or mathematical procedures involved. Such questions promote also the use of appropriate mathematical language. In episode 10, a 4th grade teacher uses a verification question («Is it 4 times 52 or 52 times 4?») to clarify the meaning of the order of the multiplication factors, reinforcing the concepts of multiplying and multiplier, and the proper use of mathematical language with understanding.

EPISODE 10

Teacher: Excuse me. Now I'm standing here thinking … is it 4 times 52 or 52 times 4?

Fábio: It is 4 times 52.

Students: No…

Rita: No, it is 52 times 4.

Teacher: What is being repeated in the cubes?

The divergency in the students' responses causes a further intervention of the teacher through a *focusing* question («What is being repeated in the cubes?»). This question directs students to the context of the mathematical task, suggesting that they reanalyse the problem. Focusing questions encourage students to return to the task, so that they can reflect on what they did, systematize what was learned, and use mathematical language appropriately.

The generalization of mathematical results is a common purpose of the systematization of mathematical learning phase, in order to construct mathematical knowledge (Canavarro *et al.*, 2012). The teachers of the multimedia cases we have used to illustrate the ideas we have put forward have significant concerns about generalization, particularly the algebraic generalization of numerical results, sometimes without resorting to algebraic notation. In the 5th grade class, the teacher tries to negotiate the generalization of mathematical results with the students, going beyond the situation of the mathematical task to other contexts, in order to enhance the students' understanding of mathematical generalization. This concern leads her to use focusing questions («We've worked with many different values, and haven't we reached the same conclusion?») centred on the mathematical solutions. The 7th grade teacher assumes that through focusing questions she will help students generalize mathematical results and make mathematical connections (episode 11).

EPISODE 11

Teacher: Exactly. So, the big difference between this strategy and this one is that if we'd change the number of votes to 7653, it'd be enough to match the first expression to 7653, whereas using the previous strategy, what would happen?

Student: We would be trying, and trying, and trying...

Teacher: Exactly, we would be here ... in a much more complicated process.

The teacher asks the students to compare two mathematical strategies, algebraic modelling through equations and recognition of an algebraic pattern using numerical sequences, as a way of systematizing mathematical learning.

In episode 12, the mathematical connection between the two strategies – equations and sequences – is addressed by the teacher with focusing questions centred on data and procedures.

EPISODE 12

Teacher: Now my question is: look now at our general terms, those of these sequences, and look at the equation Mariana and David have written.

Students: It's the same.

Teacher: Okay. In other words, using trial and error...

Student: It's the same thing...

Emphasizing the connection between different mathematical strategies allowed students to get a better grasp of the algebraic relationship of the numerical sequences underlying the situation at hand. The teacher uses a verification question to ensure that students make sense of the algebraic expression for the general term of the numerical sequence and that they know what mathematical procedures are necessary to find the order of the term: «General term, and from here, if I had 7656 votes, what would I have to do to find the order?»

Thus, the systematization of mathematical learning phase is characterized by verification questions of acquired (institutionalized) mathematical knowledge, and by focusing questions centred on situations of mathematical incorrectness or difficulties evidenced by students. This phase develops around the reanalysis of data, procedures and mathematical strategies, with the ultimate goal of systematizing mathematical learning (Stein *et al.*, 2008).

CONCLUDING REMARKS

Dialogue, both amongst students and between the teacher and the students, is a significant feature of inquiry-based mathematics teaching. Such dialogue emerges, to a great extent, from students' mathematical activity which, in turn, is based on challenging tasks posed by the teacher (Ponte, 2005; Stein *et al.*, 2008). Though dialogue may spontaneously arise among students, it can be significantly enhanced by the teacher when inviting students to participate by requesting information. This may happen at any stage of the lesson (Hufferd-Ackles *et al.*, 2004).

As we have seen, one of the main purposes of teachers' questions is to gather information they do not possess, in order to access students' knowledge and thinking (through inquiry questions) or to assess students' knowledge (via verification questions). Focusing questions, while also generating dialogue, fulfil a specific purpose. They usually lead students to rethink their oral or written answers, focusing on particular aspects that the teacher deems relevant (Mason, 2000; Nicol, 1999).

This connection between the teacher's questions and the creation of opportunities for dialogue, which is not a direct relationship, is particularly evident in the practice of inquiry-based mathematics teaching. As we advocate, and as we have tried to illustrate through episodes of mathematics lessons involving students of various grade levels, the teacher's question is a discursive act that plays a fundamental role in inquiry-based mathematics teaching (Guerreiro, 2011; Hufferd-Ackles *et al.*, 2004). Thus, the questioning of the mathematics teacher, which is seen as a professional practice, is an important and hardly replaceable piece of inquiry-based teaching. Therefore, rather than talking about *good questions* in a mathematics classroom, it is more apposite to focus on good questioning *practices*, i.e., the appropriate use of questions, in a particular context, taking into account the goals one wants to achieve (Aizikovitsh-Udi, Clarke & Star, 2013).

It is natural for the teacher to pose questions throughout the various phases of an inquiry-based mathematics lesson. These questions may be of different types (verification, focusing, and inquiry questions), whether, in each phase of the lesson, the teacher needs to check, focus, or inquire her students' mathematical knowledge. However, given the different nature of the work students do throughout an inquiry-based lesson (as opposed to lessons guided by a direct teaching approach), some types of questions may predominate during each phase.

Thus, in inquiry-based mathematics teaching, verification questions are predominant (i) at the beginning of the lesson, in the introduction of the task phase, when the teacher verifies students' mathematical knowledge and their understanding of the task; and (ii) at the end of lesson, when the teacher aims to *institutionalize* learning, ensuring that students have developed new knowledge. Although verification questions are also common in a direct teaching approach to mathematics, in an inquiry-based approach these questions aim essentially to support assessment for learning and assist the teacher in deciding what she will do next (Mason, 2010).

Focusing questions act as indirect aids for the students. They focus the students' attention on errors, misunderstandings and alternative strategies, allowing them to build on their own reasoning and develop their autonomy. Cengiz *et al*. (2011) identify *extending episodes* during a mathematics discussion when, by means of focusing questions, the discussion moves to a different mathematical idea. However, focusing questions are also important in helping students understand and connect different ways of thinking or new mathematical ideas; thus, focusing questions are relevant when systematizing mathematical learning. As such, in inquiry-based mathematics teaching, focusing questions are suitable when the teacher monitors students' autonomous work, and also when she orchestrates collective discussions and brings the whole mathematical activity to a close. Thus, a significant presence of focusing questions makes sense during all the phases of an inquiry-based lesson except the first.

Inquiry questions make particular sense during the wintermediate phases of an inquiry-based mathematics lesson, i.e., during the development and the discussion of the task phases. When students are undertaking the task, the teacher's monitoring of their work stems largely from inquiry questions, which help the teacher gain understanding of the students' thinking. When students are undertaking the task, the teacher's monitoring of their work stems largely from inquiry questions, which help the teacher in gaining understanding of students' thinking. During the discussion phase, inquiry questions are especially relevant. They trigger students' explanations and justifications, fostering

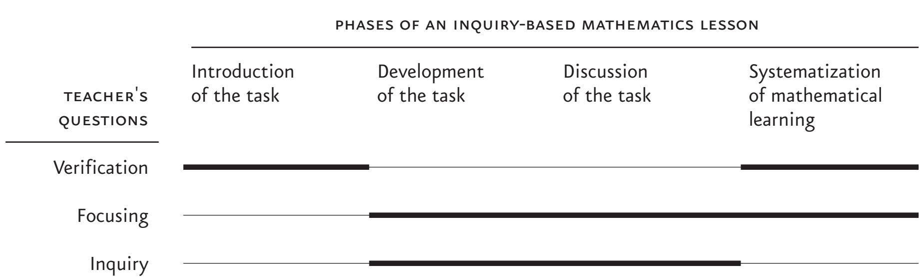

FIGURE I – THE TEACHER'S QUESTIONS IN AN INQUIRY-BASED MATHEMATICS LESSON the emergence of mathematical concepts, their terminology and their forms of representation (Ruthven *et al.*, 2011; Stein *et al.*, 2008). Figure 1 shows the phases of an inquiry-based mathematics lesson in which the different question types play a greater role.

The teacher's questions, as presented in this essay, play a central role in a mathematics teaching approach in which students develop their mathematical knowledge in interaction with one another, through negotiation of meanings, and not (exclusively) by direct transfer from the teacher.

Finally, we believe that this reflection on the role of teachers' questions in inquiry-based mathematics teaching opens several avenues for further research and poses various challenges to teacher education. The teachers in the episodes presented are experienced teachers and skilful questioners, focusing their questioning on mathematical learning. But what happens with less experienced teachers in an inquiry-based approach? Research has pointed out the significant influence of the teacher's knowledge, particularly content knowledge, in her suitable use of questions (e.g., Ball, 1991; Kahan, Cooper & Bethea, 2003; Ma, 1999; Mason, 2010). The issue rises: how does the teacher's mathematical knowledge influence her use of questions when teaching in an inquiry-based approach? These are some issues that require further research.

The complexity of inquiry-based mathematics teaching and, in particular, the role played by the teacher's questions, pose challenges to teacher education (pre and in-service teacher education). How to foster teachers' awareness of the role of questions as tools for the teaching and learning process?

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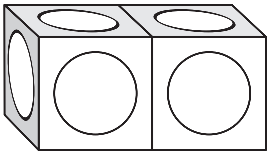
APPENDIX

TASK OF THE 4TH GRADE LESSON

CUBES WITH STICKERS

Joana is building a game with cubes and stickers. She connects the cubes through one of their faces and forms a queue of cubes. Then she glues a sticker in each of the cube's faces. The figure shows the construction that Joana did with 2 cubes. in that construction she used 10 stickers.



1. Find out how many stickers Joana used in a construction with:
1.1 three cubes 1.2 four cubes 1.3 ten cubes 1.4 fifty two cubes

2. Can you find out what is the rule that allows you to know how many stickers Joana used in a construction with any given number of cubes? Explain how you thought.

TASK OF THE 5TH GRADE LESSON

THE RISE AND FALL OF FUEL PRICES

As you probably have noticed by now, fuel prices vary a lot, according to the price of the oil barrel. Petrolex Lda. pump stations have increased the fuel price by 10%, giving rise to a choir of protests by car drivers. As a reaction, the Director of Petrolex Lda. decided to lower ce by 10%. Did the fuel price return to its previous value? Justify your answer.

TASK OF THE 7TH GRADE LESSON

THE CLASS PRESIDENT'S ELECTION

The head teacher of the class coordinated the whole process for electing the class president. After the voting process, she told the class that:

1. all 30 students in the class have voted; no blank or null votes were cast;
2. only three students received votes: Francisca, Lucas and Sandra;
3. Lucas got two less votes than Francisca;
4. Sandra got twice as many votes as Lucas'.

Who won the election? With how many votes?
Do not forget to present and explain your reasoning.