DEVELOPING KNOWLEDGE OF INQUIRY-BASED TEACHING BY ANALYSING A MULTIMEDIA CASE: ONE STUDY WITH PROSPECTIVE MATHEMATICS TEACHERS

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ABSTRACT

This study aims to understand the knowledge of inquiry-based teaching that prospective mathematics teachers reveal when analysing a multimedia case featuring one mathematics teacher´s practice in one 7th grade lesson. Data analysis focused on individual reflections that prospective teachers had written, covering four major points: the nature of the task, articulation and purpose of the lesson phases, the teacher's role in inquiry-based teaching, and anticipated challenges. The results show that the use of the multimedia case enabled prospective teachers to develop an understanding of various dimensions of inquiry teaching practice and its complexity, while they simultaneously began to predict specific difficulties they would face in their future professional practice. Significant elements of a dialogical perspective of learning also emerged from the study, as the prospective teachers recognize the central roles of language in knowledge construction, and of the interaction between teacher and student and among the students, in different moments of the lesson.

KEY WORDS

Pre-service teacher education; Inquiry-based teaching; Multimedia cases; Dialogical perspective of learning; Mathematics teaching.

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Developing Knowledge of Inquiry-Based Teaching by Analysing a Multimedia Case: One Study with Prospective Mathematics Teachers

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INTRODUCTION

Teacher education at a national and international level, coupled with innovative curriculum guidelines, has contributed to the promotion and discussion of new perspectives in mathematics teaching. In Portugal, the basic education mathematics curriculum (ME, 2007) and research into Mathematics education with projects that have focused on mathematical tasks for students, have introduced the concept of «exploratory learning» in mathematics teaching practice (Ponte, 2005). The type of teaching required is demanding, and given the limited or non-existent contact prospective teachers have had with this new reality, the complexity has become compounded. It is therefore important to provide them with observation experiences in which they can analyse the various aspects of inquiry-based teaching.

During the P3M project, multimedia cases were developed that featured inquiry-based teaching. The cases have been used in pre-service and in-service mathematics teacher education (Canavarro, Oliveira & Menezes, 2012).

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Widely used in recent years, classroom videos have been the object of a growing body of research on the implications of using them during the initial education of mathematics teachers (Climent, Romero-Cortés, Carrillo, Muñoz-Catalán & Contreras, 2013; Llinares & Valls, 2010; Santagata & Guarino, 2011). Yet none of these studies has specifically focused on inquiry-based teaching.

The authors assume that mathematics teaching practice as presented in the multimedia case takes a dialogical perspective of knowledge construction (Wells, 2004), ultimately aiming to augment the prospective teachers' understanding of this practice. Thus, with this study featuring the teaching practice of a 7th grade teacher, we intend to examine the kinds of knowledge prospective teachers reveal of the complex, and challenging practice known as inquiry-based teaching, after they have explored one multimedia case. The study was carried out in an initial teacher education course of the master's degree program in mathematics teaching at the University of Lisbon.

THEORETICAL FRAMEWORK

PERSPECTIVES OF INQUIRY-BASED TEACHING

The word *inquiry*, in the context of education, has been used for decades with various meanings that were in line with a number of theoretical approaches to learning. Many authors who propose the *inquiry-based approach* as an alternative to expository teaching (Chapman & Heater, 2010; Towers, 2010; Wells, 2004) base their theories on Dewey's perspectives of learning. This type of practice is also linked to *big ideas* such as «learner-focused, question driven, investigation/research, communication, reflection, and collaboration» (Chapman & Heater, 2010, p. 448), which played a major role in the reformist ideas of curriculum recommended by the NCTM (2000). In Portugal, some research projects into math education propounded that investigative activity be brought into the classroom, using open-ended tasks in which the students ask questions and get involved in formulating, testing and proving conjectures (Ponte, Brocardo & Oliveira, 2006).

To focus on the inquiry-based teaching approach, we took the theoretical perspective of Wells (2004) who believes that knowledge «is constructed and reconstructed between participants in specific situations, using the cultural resources at their disposal, as they work toward the collaborative achievement

of goals that emerge in the course of their activity» (p. 105). It is a perspective that views knowledge as being situated and constructed in cooperation with others, and reflecting on what has been learned.

Wells (2004) derives several implications for teaching, of which we highlight the following: i) knowledge must be constructed from problems and issues that are meaningful to the students, encouraging their understanding; (ii) it is important to develop individual autonomy and capacity for action, while stimulating interdependence and the value of collaboration; (iii) this knowledge can only be constructed from previous experiences by dealing with problems that arise in the course of specific, practical activities ; and (iv) language has a central role as it mediates knowledge in a process of meaning assignment, which is at the core of teaching and learning activities.

Wells (2004) asserts that the teaching and learning process should be seen as a process of *inquiry* that is built jointly by the teacher and the students through dialogue. He advocates a teaching practice that emphasizes the students and the conditions that favour participation in inquiry activities that are both collaborative and individual. With this perspective, what we learn is that we do and what we do depends on the practices that are available in the communities in which we participate. All of this underscores the fact that students' mathematical learning (or learning in any other area) is strongly influenced by the teaching that is going on in a given context.

In Portugal, the expressions «exploratory teaching», which describes a certain teacher practice (Canavarro, 2011) and «exploratory teaching and learning» (Ponte, 2005) have been used to describe an approach that differs starkly from directive teaching which is posited on knowledge transmission effected by the teacher who explains the content after which the students practice by applying the concepts and procedures that were taught. Exploratory teaching focuses on students' activity, through challenging tasks that allow multiple entry points while stimulating students' mathematical thinking. This approach gives students a greater degree of autonomy. In this study, we adopt the expression «inquiry-based teaching» in line with the international literature.

One of the aspects highlighted by inquiry-based teaching is the selection of tasks that engage the students in significant mathematical activity. These activities are aimed at honing their reasoning and understanding of mathematical concepts and processes. The NCTM (1994) refers to mathematically valid tasks that promote the development of mathematical understanding and skills, and encourage the establishment of connections, formulation and

solution of problems, mathematical reasoning and communication, and the elevation of mathematics learning as a permanent human activity.

Within an inquiry-based teaching framework, the mathematical tasks proposed are of particular importance. It is from them that the student's mathematical activity unfolds. Thus, the tasks assigned should allow students «to think mathematically about important ideas and assign meaning to mathematical knowledge that emerges from the collective discussion on these tasks» (Canavarro, Oliveira & Menezes, 2012, p. 256). These tasks may be problems, investigations or explorations (Ponte, 2005) but should exhibit several common traits. They should be challenging and be based on a concrete situation; allow the students to rely on their experience when solving them and, therefore, make use of various strategies with different levels of mathematical sophistication. They should be anchored in the curriculum and be aimed at a deeper understanding of mathematical concepts that have a strong connection with the knowledge students build during the lessons.

In the context of the P3M project, a framework for the practice of inquiry-based teaching (Annex 1) was designed that was based on the literature and the analysis of Portuguese teachers' practices (Canavarro, Oliveira & Menezes, 2012). This framework has a four-phase classroom structure (introduction of the task, students' autonomous work on the task, collective discussion and systematization), in which the authors identify specific actions of the teacher with two distinct but interrelated main objectives: (i) to promote the students' mathematical learning; and (ii) to manage the students, the class and the functioning of the classroom. This appears to be a fairly complete picture of what may be the teacher's deliberate actions in inquiry-based teaching, yet we do not find all these aspects in a single lesson with these characteristics. Summing up some of the most important aspects of the teacher's role during the course of this type of lesson (Canavarro, Oliveira & Menezes, 2012), we then intend to broach the various phases of the lesson.

In the *introduction of the task* phase, the teacher has to ensure that the students «own» the task. He/she must see that they are willing to take part, so that their mathematical activity can develop. He/she also has to organize the class and provide resources for the task to get done.

In the next phase, the *students work on the task* in small groups or pairs. The teacher has to ensure that the task runs smoothly, without compromising student's autonomy or lessening the task's cognitive challenge (Stein & Smith, 1998). The teacher also needs to pay particular attention to the quality

of the interaction between the students, while ensuring that they come up with materials that are suitable for presentation to the whole class. By this stage, teacher should have a good grasp of the work being done by the different groups in order to choose and sequence the solutions to be submitted to the larger group. This will be done in accordance with the criteria the teacher has defined in advance.

Then there is the *discussion of the task* as a whole group, in which previously selected solutions are presented by the students. The literature recognises this as a particularly demanding moment for the teacher in management terms, especially if the students have been working on challenging tasks (Cengiz, Kline & Grant, 2011; Scherrer & Stein, 2013). During this phase, the teacher must create and maintain an appropriate environment for presentation and discussion, by both promoting and managing student's participation. The teacher should ensure that the students listen and intervene appropriately and productively so that meaningful mathematical discourse will develop. Promoting the mathematical quality of the students' presentations is fundamental to achieving lesson's goals and furthers the students' mathematical understanding.

Finally, there is the phase in which the teacher *systematizes the key learning points* highlighted by the task, particularly points that have emerged during the discussion of the students' problem-solving strategies. Here there is more focus on the teacher. New concepts may arise or be synthesized and reviewed and procedures with which the students are already acquainted may be linked to other topics or concepts, and to transverse mathematical processes (Canavarro, Oliveira & Menezes, 2012). This is a crucial stage in which the knowledge building activity gives way to an understanding of mathematical ideas in the sense described by Wells (2004).

Within this framework of inquiry-based teaching, the teacher assumes a demanding and important role in promoting learning: from her/his choice and selection of tasks, to the structuring of the lesson and the support she/he gives to the students' mathematical activity. Inquiry-based teaching, as we understand it in this study, does not imply that the students participate in designing the curriculum by creating their own issues. Rather, it is based on a dialogical learning perspective in which knowledge is actively and collaboratively constructed by students, in environments that have been purposely created and sustained by the teacher.

THE DIALOGICAL PERSPECTIVE AND INITIAL TEACHER EDUCATION

Research has shown that the new perspectives on teaching underlying many teacher education programmes and curriculum reforms that have flourished in recent decades are very complex for prospective teachers (Lampert & Ball, 1998). These new perspectives are sometimes considered «ambitious practice» (Kazemi, Franke & Lampert, 2009), raising important questions about how to support prospective teachers to develop a professional knowledge that takes that practice as a goal. Although many prospective teachers are receptive to innovative ideas they sometimes misconstrue these ideas, and believe the myths regarding the impact such reforms have on the classroom (Oliveira & Hannula, 2008). These misconceptions are fuelled by the prospective teachers' inexperience and the lack of past exposure to such ideas and approaches (Towers, 2010).

Inquiry-based teaching has its own specificity that involves unknown dimensions for prospective teacher, who neither experienced it as students nor had many opportunities to observe it in classroom settings. Therefore, we have to create new contexts that further their grasp of this type of teaching.

Assuming a dialogical perspective regarding the knowledge for teaching (Wells, 2004), according to which the knowledge is constructed through personal experiences in progressive cycles that lead to understanding, the prospective teacher may gradually develop an understanding of inquiry-based teaching. The prospective teacher's first contact with this perspective of teaching and learning should be seen as an encounter with «information» (Wells, 2004). The theoretical ideas that are conveyed by the teacher educator or are present in the literature have to be processed and articulated through personal experience (Wells, 2002). This process of knowledge construction is essentially interactive and social, to the extent that «the individual is engaged in meaning-making with others in an attempt to extend and transform their collective understanding with respect to some aspect of a jointly undertaken activity» (Wells, 2004, p. 84); and it involves «constructing, using and progressively improving representational artefacts of various kinds» (Llinares & Valls, 2009, p. 249). Finally, understanding is seen as the culmination of this process of knowledge construction, which is action-oriented, and as a continuous process of enrichment of the personal framework from which the new experiences will be interpreted (Wells, 2004).

This model of knowledge which assumes the shape of a spiral that takes in all four quadrants (experience, information, knowledge building and under-

standing, according to Wells, 2002), enables us to sustain that the prospective teacher's understanding of inquiry-based teaching will develop at different moments, arise from multiple experiences, and be enriched by interaction with others.

In a study conducted with prospective teachers, Llinares and Valls (2009) maintain that the prospective teachers built up knowledge of mathematics teaching as a result of the arguments they put forward while interacting with their colleagues. These authors witnessed the emergence of reifications that became the subjects of discussion regarding the lessons they had observed that contributed to expand their knowledge about teaching.

This topic is also featured in a study by Towers (2010), who presents the case of a teacher, starting his career, who had difficulty in describing his vision of inquiry-based teaching to his peers. Although he put various aspects of this type of teaching into practice – for example, by promoting student's participation in large and small groups, demonstrating interest in student's alternative strategies and fostering their mathematical thinking –, he was ultimately unable to articulate his ideas, which made it difficult to collaborate with other teachers who did not share his vision of teaching.

MULTIMEDIA CASES IN INITIAL TEACHER EDUCATION

The use of videos and other multimedia resources in the classroom has become increasingly more frequent in initial teacher education. On the one hand, it compensates for the prospective teachers' lack of contact with in-class practice, and enables them to deepen their knowledge of teaching. On the other hand, it encourages the development of analytical skills targeting classroom practice, which is considered relevant to the growth of a professional teaching perspective (Koc, Peker & Osmanoglu, 2009; Stürmer, Königs & Seidel, 2013).

The video is a powerful resource that conveys a realistic image of the classroom. It contains real images of the students and teachers, and captures the voices, body language and environment of the classroom (McGraw, Lynch, Koc, Budak & Brown, 2007). By analysing a teaching and/or learning situation, prospective teachers may focus on a number of target points such as the students' thinking, the teachers' role or classroom discourse (Alsawaie & Alghazo, 2010; Koc, Peker & Osmanoglu, 2009; McGraw *et al.*, 2007). However, the focus can also be directed toward a particular curriculum topic which enhances mathematical knowledge for teaching, or toward

principles associated with pedagogical knowledge (Climent *et al.*, 2013; Seidel, Blomberg & Renkl, 2013).

Many teacher education courses integrate other resources, such as theoretical elements, interviews with teachers, resources used in lessons, students' productions, etc., which can be accessed electronically at any moment (Llinares & Valls, 2010; McGraw *et al.*, 2007). With this diversity of elements, one can design multimedia classroom cases that encompass the complexity and the different strands of teacher practice, include information of different types, and in some studies, be taught in tandem with virtual discussion settings (Koc, Peker & Osmanoglu, 2009; Llinares & Valls, 2010). These contexts have proved to be instrumental in building bridges between theoretical and practical knowledge. They allow the prospective teachers to develop a reference framework for analysing observed practice and a «metalanguage» to discuss it with their peers (Llinares & Valls, 2010).

The multimedia case in this study illustrates the inquiry-based teaching practice of a 7th grade teacher that unfolds from a mathematical task entitled «Election of the Class Representative» (Annex 2) (Oliveira, Canavarro & Menezes, 2012a). The case exposition has a narrative structure, as it contains a sequential analysis of the lesson and its preparation, starting with the task selected and the lesson plan designed by the teacher. It also discusses the teacher's intentions with regard to each phase of the lesson (introduction, students' autonomous work, discussion and systematization).

For each phase of the lesson, video segments are presented, accompanied by a transcription of teacher and student's interventions. Questions are asked to help prospective teachers examine particular aspects of the teacher's actions that seek to promote learning and the classroom management. The student's solutions, and the teacher's analyses and explanations about those are also available.

Finally, the prospective teachers are asked to focus on the teacher's reflections within the framework, «Intentional Actions of the Teacher in an Inquiry-Based Classroom» (Annex 1). Prospective teachers also use this framework to retrospectively examine the analysis of the teacher's practice that they did throughout the case, particularly to identify points that were included or omitted.

Along the multimedia case, small text excerpts, called «Synthesizing» are presented, enabling prospective teachers to systematise the main ideas the authors wish convey with regard to the teacher's role during each phase of the

lesson. Additionally, the website on which the case has been posted contains suggestions of readings on inquiry-based teaching.

We chose this particular teacher for this multimedia case because she consistently espouses inquiry-based teaching. The case also includes interviews with the teacher about the lesson, in which she explains the reasons behind her choices and the doubts and difficulties she faces. By including these details, we hope to provide prospective teachers with further insights into authentic teaching practice (Oliveira, Menezes & Canavarro, 2012b). This multimedia case is, therefore, a contextualized narrative that aims to be an instance of more general classes of ideas about inquiry-based teaching hence allowing multiple readings and interpretations (McGraw *et al.*, 2007).

The lesson featured takes place during the last part of the topic «First Degree Equations». According to a perspective of algebraic thinking development, the teacher hopes that the students, through the solution and discussion of the task, can apply and systematize the knowledge they have acquired and establish connections between topics in algebra (Oliveira, Canavarro & Menezes, 2012a). Since this was the first year the students worked with algebraic language and solved equations, this task was challenging for the students both in interpreting the situation and in the solving process.

The work on the multimedia case formed one of the modules in the course on Mathematics Teaching Methods, which targets prospective mathematics teachers. It is given during the 3rd semester of the master's in Mathematics Teaching for middle and secondary school. The module was designed by the two authors of this study. One is the teacher of the master's course, while the other is exclusively a classroom researcher. However, during the whole process, there was close collaboration between both authors, and significant time spent on discussing how the work was progressing. Student evaluation was done by the course instructor.

The multimedia case was analysed in the classroom in four sessions of 2.5 hours apiece over the course of two weeks. The prospective teachers worked in pairs or groups of three, sharing a computer. Assuming the dialogical perspective of knowledge construction (Wells, 2004), the group was encouraged to read and interpret the material as autonomously as possible and to dispel their doubts by asking each other for clarification. However, they were free to seek the teacher's support on details regarding the case content.

In general, each exploration session began with a brief reference, by the teacher, to the written work done by the different groups in the previous

session. The intention was to provide feedback on how this matched established objectives. Using a dialogical perspective of knowledge construction, although no collective discussion had taken place, prospective teachers, in small groups, discussed the ideas and negotiated what they would present as a written product of their work in each session. Analysis of the multimedia case assumed inquiry-based characteristics as well, since the questions posed were predominantly of an open-ended nature.

As far as working methodology was concerned, we must also emphasize the prominent role writing played, both as a product of small group work, and the written expression of individual reflections after these working sessions. This writing option is consistent with Wells' (2002) dialogic perspective, since according to this author, the construction of knowledge also occurs by means of writing. It is a dual dialogue: between the one who writes and the audience to whom it is addressed, and between the person himself and the text that emerges.

METHODOLOGY

The study took place within the framework of a broader Design Research project. Design Research involves a family of methodological approaches in which the research and development are mutually dependent (Cobb, Zhao & Dean, 2009). The module orienting the present study comprises experimentation with multimedia cases in teacher education that were built on research into inquiry-based teaching. The research reported in this paper focuses particularly on one of the aspects of learning targeted by this teacher education setting: the development of discourse on inquiry-based teaching.

Concerning data collection methods, we opted to focus on the learning that the prospective teachers displayed in the reflections they wrote on inquiry-based teaching. We did so because, as Wells, we believe that writing «encourages one to interrogate one's interpretations of others' utterances as well as of one's own personal experiences and beliefs in order to add to the ongoing dialogue» (2004, p. 129).

The class consisted of ten prospective mathematics teachers, the great majority of whom had no classroom teaching experience. Two had taught other subjects, while one had taught in higher education. Some had experience in individual remedial work or in small groups, giving private lessons.

For ethical reasons, one of the prospective teachers has not been included in this study, since the first author was also supervising his teaching practice.

For the data analysis, four major dimensions were defined: the nature of the task, articulation and purpose of each lesson phase, the teacher's role in accordance with the Intentional Actions' Table, and anticipated difficulties.

The first issue, the nature of the task, was not explicitly discussed with the prospective teachers at the multimedia case, since it is a subject that is addressed throughout the course and in others that preceded it. The issue also arises in some of the texts listed as additional readings for the multimedia case (for example, Ponte, 2005), as well as in the Synthetizing section dealing with introduction of the task, on which the prospective teachers based themselves when they were completing their individual assessment work. Knowing the importance of the task within the framework of this type of teaching, through this dimension of analysis, we aim to understand to what extent the prospective teachers master this topic, namely by identifying specific characteristics of the proposed task in the case.

The phases in an inquiry-based lesson are one of the constructs highlighted and the multimedia case examination was clearly structured by the lesson phases. As a result of their work the prospective teachers were able to clearly distinguish the four phases and what they were composed of. However, it is also crucial that prospective teachers realize how the phases are articulated, how they contribute to the mathematical goals of the class (Canavarro, 2011), and that they do not develop a compartmentalized view of this type of practice.

The examination of the multimedia case in the course focuses strongly on the analysis of the teacher's practice, particularly in her role in the promotion of learning and in the lesson's management. Therefore, the teacher's role was also an object of analysis. The Framework for the Teacher's Intentional Actions (Annex 1) is an organizing tool specifically designed for prospective teachers to be able to analyse the teacher's practice in the final phase of exploration of the case. Therefore, we wanted to understand to what extent it becomes a reference to prospective teachers when they make a more global reflection on their experience in analysing an inquiry-based lesson.

Finally, we propose ourselves to analyse the challenges posed by inquiry-based practice, particularly those prospective teachers might face when they start teaching. This is important since research has demonstrated that both knowledge and disposition or belief about teaching should be taken into account in initial teacher education (Beswick, Callingham & Watson, 2012; Oliveira & Hannula, 2008).

For each of the dimensions we set up categories from the data analysis that we summarised in tables, naming the prospective teachers that they are associated with. We present illustrative examples of these categories, using excerpts from the prospective teachers' written reflections.

RESULTS

The results of the study are organized into four sections according to the dimensions mentioned. Throughout, we sought to characterize the knowledge of inquiry-based teaching the prospective teachers demonstrated.

THE NATURE OF THE TASK

The nature of the task that is presented in the multimedia case was not explicitly discussed in our sessions. However, the prospective teachers did discuss the topic in their written reflections, highlighting its importance in this type of teaching, and identifying specific characteristics of the proposed task.

> This is, therefore, an example of a lesson in which the idea of inquiry-based teaching of Mathematics was put into practice, suggesting students can work on interesting tasks, creating their own strategies and constructing knowledge in a way that highlights the need or benefit of a particular idea, concept or mathematical procedure (Simone).

> I think we should present several tasks, some of a more closed nature and others that lead students to explore, conjecture and discover for themselves, always with the teacher's support. The tasks should also enable students to progressively use symbolic notation and appropriate forms of representation. This multimedia case was a good example of a task with the features that I listed earlier (Vânia).

The prospective teachers highlighted that the task proposed in the multimedia case enabled the students: to submit an original answer, by resorting to previous knowledge and experiences; create their own strategies for solution; build knowledge that reveals the need or the advantage of an idea, concept or procedure; develop autonomy to explore, conjecture and discover; use progressively appropriate symbolic representations and notations; and develop their mathematical thinking.

The task worked in the multimedia case was included in the didactics unit that the students were working on. While the teacher worked through the Algebra theme, she displayed a marked intention to promote the students' algebraic thinking (ME, 2007). The prospective teachers recognized the task's potential to enhance the students' mathematics skills.

> (...) the task requires the students to be able to interpret and represent a contextualised situation, using algebraic language and procedures. It also requires them to solve problems, communicate, reason and shape the situation by resorting to mathematical concepts (equations, sequences ...) (Matilde).

The prospective teachers acknowledged that the task allowed students to interpret, represent, reason and solve a problematic situation using algebraic language and procedures. They realized that the work involving regularities, which had been dealt with before, allowed the students to establish a connection with equations. The fact that the students resorted to trial and error strategies, sequences and regularities and the construction of tables, enabled the teacher to establish connections between the different strategies adopted, and to promote the development of the students' algebraic thinking, thus deepening the study of algebraic relations and their symbolization, which is essential for the development of the notion of variables and the understanding of algebraic language (ME, 2007).

In Table 1 we present a summary of the characteristics of the task, as highlighted by the prospective teachers:

THE TASK	PROSPECTIVE TEACHERS
It is not a simple exercise in which the students have to apply previously acquired knowledge; it allows for different solution strategies.	Antónia, Lourenço, Margarida, Simone, Vânia, Bárbara
It has the potential to develop skills in students: problem-solving, mathematical thinking and mathematical communication.	Antónia, Lourenço, Matilde, Vânia, Sílvio, Simone
It enables the development of the students' algebraic language and thinking.	Antónia, Matilde, Vânia, Sílvio, Simone
It promotes the understanding of the need or benefit of a particular idea, concept or mathematical procedure.	Lourenço, Margarida, Matilde, Simone

TABLE 1 – TASK CHARACTERISTICS

Most of the prospective teachers refer to at least two characteristics of the task dealt with in the lesson, although some of them also reported on the mathematical tasks within a framework of inquiry-based teaching in a more general way. Only one prospective teacher (Sandra) does not mention this matter in her reflection and another (Bárbara) refers to only one of these aspects.

THE LESSON PHASES

One of the constructs that was highlighted the most was the definition of the four phases associated with the task because, as we have mentioned, the structuring of the case was clearly oriented by the different phases. However, in addition to recognizing and distinguishing the phases, prospective teachers were able to see how they are articulated and contribute to the mathematical goals of the lesson, and thus the development of algebraic thinking.

The prospective teachers said that in the introduction phase the students had time to take ownership of the task, understand its goals and organize themselves to work. According to them, this allowed the students to interpret the task correctly, clarify doubts about language, and thus be able to distinguish what information they should retain and use for the solution.

In the prospective teachers' point of view, the presentation phase is also crucial to ensuring that the students are engaged, so that they feel challenged and confident enough to go on to the subsequent activity.

> It is important to challenge them, so that the students take ownership of the task with enthusiasm and curiosity, committing themselves to solving it. It is also the moment when the class organizes their work, defines timing, and manages resources and classroom working methods (Lourenço).

With regard to the second phase of inquiry-based teaching, the prospective teachers acknowledged that the main objective was for the students to work autonomously to complete the task. They could see the videotaped students had been given the opportunity to develop their reasoning, discuss their ideas, explain and defend their ways of thinking, question the solutions presented by their classmates, use different records, explore their findings and acknowledge their mistakes and difficulties. They also recognized how important this moment was in building the students' skill at mathematical communication.

> (...) this moment proved to be important to the extent that the students were able to make different representations, discuss their ideas, explain their reasoning aloud to classmates, highlight different solution strategies, understand the reasoning of others, explore their discoveries, deal with their mistakes and difficulties and write their conclusions (Bárbara).

Referring specifically to the linkage between the different phases of the lesson, one of the prospective teachers mentioned that discussion of the task depends to a large degree on the activity that occurred in the previous phase.

> (...) the four phases are related, each one depends strongly on what they achieved, during the discussion held in the previous phase. For example, in my view, there is a strong relationship between the phase of students' autonomous work and the segments devoted to discussion and systematization, because, while during the phase of autonomous work students explore different strategies, and the students' solutions are selected and sequenced by the teacher, in the discussion, those same ideas are discussed collectively (Bárbara).

According to one of the prospective teachers, interaction among the students while they are working autonomously allows them to feel safe enough to share their solutions, voice their doubts and question their classmates' strategies during the discussion.

> Another thing that stood out in this case was the interaction with the dyads when they were completing the task. I consider the simple act of interacting as a way of helping the student believe in himself and feel safe. (...) During this time of collective discussion, the students had the opportunity to present their strategies, share their doubts and question the strategies presented by their classmates (Antónia).

The prospective teachers saw that in the task discussion phase the students were able to understand the differences between the various strategies presented, in particular, their mathematical effectiveness. They became involved in processes of explanation and justification and developed their mathematical communication skills. The following comment also shows that this prospective teacher understood how discussion of strategies can contribute to the systematization of learning:

> In this exploratory work, by trying out different solutions during their discussions, the students were also able to see that some ways of tackling the solution are more effective than others (Margarida).

They also believed that discussion of strategies had contributed to the systematization of mathematical learning, because students had been able to give meaning to the mathematical concepts being introduced or reviewed, establish connections and formalize and generalize the concepts in question.

> (...) the students [could] cement, clarify their ideas and establish connections between the various strategies (...) Thus, I believe that this phase has helped the students to have a global view of what they have learned and the different strategies that they could have chosen to solve the problem (Bárbara).

From the previous quotations, we can see that the prospective teachers were able to recognize specific student actions and reactions that occurred during each one of the phases (Table 2). The majority of prospective teachers highlighted eight or more student actions during the different phases. Three of the prospective teachers (Bárbara, Matilde, and Vânia) identified more than ten actions. Just one prospective teacher (Sandra) only recorded two actions. However, as we have seen, they also acknowledged the existence of different relationships between the lesson phases.

THE TEACHER'S ROLE IN AN INQUIRY-BASED LESSON

The prospective teachers linked some of the videotaped teacher's actions and intentions to each phase of this inquiry based teaching lesson, and interpreted the role of her actions in promoting lesson management and student learning.

The prospective teachers picked up on the fact that, during the introductory phase, the teacher had sought to ensure that the pupils understood the task. She read the task out to the students, discussed the meaning of expressions and concepts that could have caused confusion, established connections with previous experiences and made sure that all the students were on board with the task. They also grasped that the teacher had ensured that the pupils understood what was expected of them, informed them of how long they had, and offered resources that could help them complete their task.

PHASES OF LESSON	THE STUDENTS HAD THE OPPORTUNITY TO:	PROSPECTIVE TEACHERS
Introduction of the task	- take ownership of the task, and understand its goals	Bárbara, Lourenço, Margarida, Matilde, Vânia
	- organize the work	Lourenço, Matilde, Simone
	- correctly interpret the wording of the task and distinguish important information to retain and use	Bárbara, Margarida, Matilde, Simone, Sílvio
	- clarify doubts regarding language	Bárbara, Margarida, Sílvio, Vânia
	- be challenged	Lourenço, Sílvio
Students' autonomous work	- be autonomous	Antónia, Matilde, Sandra, Sílvio, Simone, Vânia
	- discuss ideas	Bárbara, Matilde, Vânia
	- explain and defend their ways of thinking, exploring their findings	Antónia, Bárbara, Matilde
	- question the solutions presented by classmates	Antónia, Bárbara, Matilde
	- use different records	Bárbara, Matilde, Vânia, Margarida
	- highlight different solution strategies	Antónia, Bárbara, Vânia
	- understand the reasoning of others	Bárbara, Margarida
	- deal with their mistakes and difficulties	Antónia, Bárbara, Matilde
	- formalize their conclusions	Bárbara, Vânia
Discussion of the task	- understand the differences between the various solution strategies and records	Bárbara, Lourenço, Margarida, Matilde
	- present solutions and justifications	Bárbara, Matilde, Vânia, Simone, Sílvio
	- share their doubts	Antónia, Simone
	- analyse the effectiveness of strategies and records	Bárbara, Lourenço, Margarida, Simone
	- understand concepts, processes and procedures	Bárbara, Lourenço, Margarida, Simone, Sílvio
Systematization	- give meaning to mathematical concepts presented or reviewed during the completion of the task	Bárbara, Sílvio, Vânia
	- establish a connection between the different strategies and concepts	Bárbara, Matilde, Simone, Vânia
	- record conclusions, definitions and properties	Sandra, Simone
	- accomplish learning	Bárbara, Matilde, Sílvio

TABLE 2 – STUDENTS ACTIONS IN THE LESSON PHASES IDENTIFIED BY PROSPECTIVE TEACHERS

> One of the teacher's concerns was to ensure that there were no language issues, in particular regarding the meaning of blank and null votes. Finally, she defined the working methodology needed to carry out the task, taking into account the characteristics of the room and the students' behaviour. She reminded the students that they would have to present their rationales. She also provided material (Vânia).

The prospective teachers highlighted the ways in which the videotaped teacher made sure that the task was completed. She gave students the space to develop their own solution strategies; monitored them and gave them feedback, mindful not to change the level of the task's cognitive demand; helped students who were having more difficulty or were more uninterested, without giving them the answer; challenged the students who were more advanced; encouraged the use of mathematical communication in the classroom; furthered the students' skill at argumentation; tried to interpret and understand how the students had solved the tasks; identified unexpected responses and errors; identified the potential of the strategies used by the students and supported them in the strategies they had chosen; and matched the students' written work with the expectations she had lined out when planning the task, so that she could organise the sequence in which the students presented their solutions during the discussion (third phase).

> Claudia, the teacher, always supported students in the strategy that they had chosen, not influencing them to take a certain path. This teacher was also very careful never to validate the students' mathematical output, in order not to dash their expectations regarding the work they had already done or were about to do. However, it is crucial for the teacher to understand the mathematics of her students, who sometimes, despite being correct, go in a different direction than the teacher expected/planned (Antónia).

The prospective teachers stressed that during the discussion phase, the teacher had been committed to making sure that the pairs shared and compared several solution strategies. She also made it clear that there is no one single strategy that is ideal and infallible. The future teachers also noted that she had analysed the potential of their strategies and discussed their weaknesses, so that the students could learn – not only from the activity itself – but also from the reflections about it. In addition, as they noted, she articulated the students' ideas of what they were expected to learn; discussed the mathematical learning involved in

the task; and established connections between the various mathematical topics involved.

> The teachers' main concern should be to promote mathematical communication, through reflection and instruction because this is how enriching learning experiences emerge. It highlights how positive questioning can be since it focuses the students' attention on the task and helps them to make their reasoning explicit. Encouraging communication, the explanation of ideas and questioning among classmates, and not just between the teacher and the class (Sandra).

This phase contributed to the accomplishment of some of the lesson's mathematical goals. The presentation of different equations for the same problem allowed the students to discuss whether or not the equations were equivalent. The students also had the opportunity to establish connections between different types of mathematical records and to see the different strategies they could use to solve the problem.

> One particularity was that there were groups with different equations for the problem. The teacher explored the idea that these equations were not all the same because the variable represented different things in different cases (Simone).

According to the prospective teachers, during the systematization phase, the teacher assumed the role of protagonist, formalizing concepts, ideas and procedures related to the topic and focusing on the transverse skills that had been highlighted during the discussion phase. The teacher revisited concepts learned during the teaching unit, established connections with previous learning, summarized the conclusions reached during the task and guaranteed that the main ideas were written down.

Table 3 shows the videotaped teacher's actions, during the lesson's four phases, that the prospective teachers noted. Seven prospective teachers refer to at least eight actions that typify the teacher's role within a framework of inquiry-based teaching. However, there is one prospective teacher (Matilde) who only mentions two aspects.

PHASES OF LESSON	THE TEACHER SOUGHT:	PROSPECTIVE TEACHERS
Introduction of the task	- to ensure the understanding of the task and that all students joined in	Bárbara, Lourenço, Simone
	- to discuss the significance of the task's expressions and concepts	Bárbara, Lourenço, Vânia
	- to establish connections with students' previous experiences	Simone
	- to guide the class organization	Bárbara, Lourenço, Simone, Vânia
	- to offer instruments/records that assist the students in solving the task	Lourenço, Margarida, Vânia
Students' autonomous work	- to promote autonomy	Lourenço, Margarida, Sandra, Simone, Vânia
	- to monitor and give feedback to students during the interactions	Bárbara, Sandra, Simone, Vânia
	- to assist students with difficulties	Margarida
	- to challenge the students	Bárbara, Margarida, Simone, Vânia
	- to promote communication in the classroom	Bárbara, Lourenço, Simone, Vânia
	- to interpret and understand how students solved the task	Antónia, Vânia, Sandra
	- to identify the students' unexpected responses and errors	Antónia, Margarida
	- to identify the potential of the students' strategies and to support them in developing them	Antónia, Bárbara
	- to compare the students' written work with the expectations that she had when the task was planned	Antónia, Margarida
Discussion of the task	- to provide the sharing of several strategies and mutual respect with regard to their classmates' explanations	Antónia, Lourenço, Sandra, Sílvio
	- to define the order in which the solutions would be presented	Lourenço, Margarida, Sílvio, Simone
	- to design questions that promote student reflection and to assist them in clarifying their ideas, without validating or discrediting their work	Lourenço, Margarida, Sandra, Sílvio, Simone
	- to compare solutions, analyse their potential and discuss their weaknesses	Lourenço, Margarida, Sandra, Sílvio, Simone
	- to approximate and articulate the students' ideas	Antónia, Sandra
Systematization	- to formalize concepts, ideas and procedures	Antónia, Matilde, Simone, Vânia
	- to foster transverse skills	Simone
	- to establish connections between the types of learning targeted	Antónia, Bárbara
	- to ensure that the main ideas are written down	Lourenço, Matilde, Simone

TABLE 3 – TEACHER'S ACTIONS IN THE COURSE OF THE FOUR PHASES OF THE LESSON, AS NOTED BY THE PROSPECTIVE TEACHERS

ANTICIPATING CHALLENGES

The multimedia case led prospective teachers to reflect on the challenges that arise in inquiry-based teaching, particularly those that they may experience – or have already experienced – in their own teaching. They stressed that, due to their lack of experience, they would find it hard to design interesting and productive tasks; to anticipate different solutions the students can come up with and how to link those solutions to the mathematical goals that had been set; to establish connections among mathematical ideas; to select student strategies and sequence and manage the discussion among students.

Other classroom management issues the prospective identified as potentially challenging were: how to manage time, student interaction, certain events that occur simultaneously in the classroom and discussions that involve decision-making and affect the students' learning.

> Regarding difficulties (...) I identified three that seem relevant:
> – Being able to reach all students and pay attention to various aspects that occur simultaneously in the classroom;
> – Making decisions that can affect [students'] learning opportunities. For example, in one phase of the lesson, the teacher decided to go beyond the time she had planned and justified her decision by the richness and diversity of strategies that emerged;
> – «Orchestrating» the discussion in large groups and promoting student reflection. This is one of the roles that I find is frequently referred to as being one of the most difficult for the teacher (Lourenço).

To encourage students to reflect, and to maintain the level of cognitive demand posed by the task, to explore their thoughts, poll and give meaning to their ideas, to design questions that enable students to progress without giving answers were the points prospective teachers deemed as potentially challenging in this type of teaching.

> Other difficulties that may arise are managing the interactions among students; getting all the groups to work and seeing that within each group all the students work; ensuring that the students do not digress but keep focused during the solution; and being able to resist giving direct answers to the students (Margarida).

The following table lists the challenges of inquiry-based teaching, as noted by prospective teachers. Most of them describe four or more aspects. Two prospective teachers made no reference to this topic in their reflections.

ANTICIPATED CHALLENGES:	PROSPECTIVE TEACHERS
– finding interesting , enriching tasks to create learning opportunities	Margarida, Sílvio, Vânia
– anticipating different student strategies	Sílvio
– relating the solutions to the mathematical purpose of the lesson	Sílvio, Vânia
– establishing connections between mathematical ideas	Bárbara, Matilde
– selecting the strategies and sequencing them	Matilde
– managing the discussion so that everyone participates	Lourenço, Margarida, Matilde, Sílvio, Simone
– managing interactions among the students	Lourenço, Margarida, Simone
– preparing questions for the students to think about, so that they can accomplish their tasks	Bárbara, Margarida, Matilde, Vânia
– managing time	Lourenço, Vânia

TABLE 4 – CHALLENGES OF INQUIRY-BASED TEACHING ANTICIPATED BY PROSPECTIVE TEACHERS

DISCUSSION AND CONCLUSIONS

Throughout this study, we have sought to understand the knowledge prospective teachers display regarding inquiry-based teaching. We have based our work on their analyses of a multimedia case featuring a teacher teaching a 7^{th} grade mathematics lesson. We identified four important dimensions that enabled us to examine how this knowledge was expressed in the prospective teachers' written reflections.

Realizing how important the first dimension – the nature of the task – is to this type of teaching, we tried to judge to what extent the prospective teachers highlight this topic and identify the specific characteristics of the task in the case. Although at the outset, this task could have been construed merely as an attempt to solve an equation, the prospective teachers were able to discern its contribution to achieving the mathematical goals of the lesson. They were also able to relate it to inquiry-based teaching practice. They acknowledged that the task had given the students the freedom to create their own strategies and assess the need for or the advantage of a particular mathematical idea, concept or procedure. They recognised that it had also helped

students develop their mathematical skills, in particular algebraic thinking, mathematical reasoning and mathematical communication.

Understanding and reflecting on the task's role enables prospective teachers to choose tasks that suit their teaching goals and prioritize them according to their potential to trigger complex ways of thinking that help students establish connections to meanings or to ideas and mathematical concepts (Cyrino & Jesus, to appear). However, since not all prospective teachers elaborated on this topic sufficiently in their reflections, we did not obtain a complete picture of the knowledge that the whole group had acquired on this aspect of an inquiry-based teaching.

Most of the prospective teachers concluded that the mathematical task does not guarantee a significant mathematical activity by itself. They have overcome a naive vision that in order to transform the mathematics teaching, it is enough to propose good tasks. They very obviously came to realise that learning indeed occurs through enriching mathematical activities, but that is necessary that students reflect on these activities, an idea that this was well illustrated in the way the videotaped teacher organised the lesson and in her oral observations about it.

The multimedia case we used was clearly driven by the different *phases of an inquiry-based lesson* (introduction, implementation, discussion and systematization). The prospective teachers were able to recognize and distinguish between each of the phases, realizing that they articulate and complement each other in developing the students' algebraic thinking, problem-solving skills, mathematical reasoning and communication. This recognition is fundamental if prospective teachers are to understand the advantages of this type of lesson structure, and see that it is not rigid.

Thus, it is significant that the prospective teachers were able to link specific actions and/or features in the students' learning to each phase of the lesson, noticing that the students' role is front and centre in this type of teaching, although this is not the analytical perspective adopted in the multimedia case which focuses on the teacher's intentional actions (Annex 1). We see, with their comments, the elements of a dialogical learning perspective (Wells, 2004) in which language is assumed to play a central role in the knowledge processes, in the interaction between teacher and students, and among the students, throughout the various phases of the lesson.

Regarding the teacher's role in inquiry-based teaching, prospective teachers pinpointed several teacher actions belonging to each lesson phase of the

videotaped class. They emphasized aspects of teaching practice such as questioning in order to understand the students' thinking and promoting communication among students, which showed that they were picking up on the dialogical perspective of learning.

In the last section of the multimedia case, we presented the framework entitled «Intentional Actions of the Teacher in Inquiry-Based Teaching» (Annex 1). This framework has become an important tool for prospective teachers as far as the two major dimensions of teaching practice are concerned – promoting learning and classroom management. It has also served to clarify the various approaches that the phases encapsulate. It seems quite significant that, as prospective teachers, they chose to talk about the teacher's role in inquiry-based teaching in a way that was not restricted to the ideas and vocabulary that were presented in the framework and in a way that cogently covers the various phases of the lesson.

Finally, when it came to identifying the challenges of inquiry-based teaching practice, it appears that the majority of prospective teachers did anticipate many of the difficulties that may arise. They refer to aspects that show that they recognise the characteristics of this type of teaching and the actions required of the teacher. Among the challenges they cite are: choice of appropriate tasks; classroom management (including communication); dealing with the student ideas and output through appropriate questioning, while not limiting their mathematical activity; and establishing connections among mathematical ideas. It should be noted that most of the prospective teachers went beyond the videotaped lesson being analysed and were able to anticipate the challenges on a more general level. Even so, there were two students who did not mention any type of challenge. This lack of foresight is critical because, without the ability to predict, the future teacher will not be able to problematize. By anticipating challenges, beginning teachers are projecting themselves into the teaching role while developing a more realistic view of this type of teaching, which is indeed complex.

By watching the lesson featured, the prospective teachers were given access, to a type of inquiry-based teaching they were not familiar with because they had not experienced it as students, nor had they been exposed to it before in their teacher education programme. Knowledge is constructed from personal experiences in progressive cycles that lead to understanding (Wells, 2004). Hence, exploration of the multimedia-based lesson proved to be very promising because firstly, it gave the student teachers access to a body of

information, by means of successive approximations. It enabled them to witness the classroom experiences of both a teacher and her students in a way that embodied the theory. The construction of knowledge was intensified, since the prospective teachers became involved in the production of meanings; constructing and reconstructing collective concepts about the nature of the task by working among themselves and in conjunction with the teacher educator; understanding and articulating the purpose behind the lesson phases; grasping the nature of the teacher's role within the framework of inquiry-based teaching; and fathoming the complexity associated with this type of practice. This process of construction of knowledge was essentially social and interactive; and this enabled them to engage in a type of discourse about inquiry-based teaching, an approach that is considered to be complex and challenging for prospective teachers.

By viewing real videotaped classroom material, prospective teachers improve their ability to interpret the overarching topics and nuances of teaching, and get to see how much the teacher's actions impact classroom dynamics and the students' learning (Alsawaie & Alghazo, 2010). The opportunity to witness relevant teaching practice plays an important role in the development of their knowledge about teaching; the same has been true in other studies where classroom video analysis has been used (Koc, Peker & Osmanoglu, 2009; Stürmer, Königs & Seidel, 2013).

In their written reflections, the prospective teachers often refer to particular elements in the multimedia case that illustrate or substantiate statements about inquiry-based teaching. This reinforces the idea that the knowledge that they are developing about this type of teaching is situated, although it may represent more extensive classes of ideas (McGraw *et al.*, 2007). In this multimedia case the theory was not presented in the beginning. It was, rather, built from the analyses that were done. In the final phase, when the students already had the reference framework, Intentional Actions of the Teacher in Inquiry-Based Teaching, in their possession, they were able to review the practice having observed the theory in action, discuss it, and attribute meaning. The overall comprehension of inquiry-based teaching that they demonstrated in the reflective essay they wrote afterwards showed that they had taken in the core features of inquiry-based teaching. However, the role of theory in analysing or reflecting on a situation or observed practice is not linear (Llinares & Valls, 2010), which is why we need to further explore the issue in future research on the topic.

In conclusion, our study shows that, in general, the prospective teachers who participated indeed gained knowledge of inquiry-based teaching, recognizing its main characteristics and potential, and the challenges it may pose. The study highlights the potential such a teacher education setting has for the development of a dialogical perspective of learning by prospective teachers. In the study, they acknowledged the central role language plays in the knowledge processes, in the interaction between teachers and students, and among the students themselves. They also highlighted the role of teacher questioning in understanding the students' thinking, and in promoting communication.

Using this study as a departure point, it would be interesting to embark on a more individualized analysis, using other data sources, so that we might develop an even deeper understanding of the knowledge each prospective teacher gains with regard to inquiry-based teaching. This analysis and further research will enable us to determine what areas of this teacher education setting should be invested in and which should be reformulated, so that prospective teachers come to realise that this complex teaching approach is not only the bailiwick of a few expert teachers (Kazemi, Franke & Lampert, 2009; Santagata & Guarino, 2011). Future research should attempt to unveil how prospective teachers integrate these ideas into the next phase of teacher education – their supervised teaching practice – which occurs in different contexts, some of which are more conducive to inquiry-based teaching than others.

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	PROMOTION OF MATHEMATICS LEARNING	CLASS MANAGEMENT
Introduction of the task	*Guarantee the appropriation of the task by the students by:* – clarifying unfamiliar vocabulary – mobilizing and verifying prior knowledge – setting goals *Promote students' commitment to the task by:* – challenging them to work – establishing connections to students' prior experiences	*Organize the students' work by:* – establishing the time for each phase of the lesson – establishing the organizational structure of the work (individual, pairs, small groups, whole-class) – organizing classroom materials
Students' work on the task	Ensure the students carry the task through by: – posing questions and giving clues – suggesting representations – focusing on productive ideas – requesting clarification and justifications *Keep the cognitive challenge by:* – promoting student's reasoning – trying not to validate the mathematical correctness of students' answers	*Promote the work of students/groups by:* – setting up interaction among students – providing materials *Guarantee the production of materials for students presentations by:* – requesting their records – providing appropriate materials – setting aside time to plan the presentation *Consider the selection and sequencing of students presentations by:* – identifying solutions that are more or less comprehensive, complete and/or formal – identifying solutions with common errors – sequencing the solutions selected
Discussion of the task	*Promote the mathematical quality of the presentations by:* – asking for clear explanations with mathematical evidence – asking for justifications of the outcomes and representation used – discussing the difference and the efficacy of the solutions presented. *Promote interaction among students in the discussion of mathematical ideas by:* – encouraging questioning for the clarification of ideas – encouraging analysis, debate and comparison of ideas – identifying and making available to discuss questions or mistakes in the presentations	*Create a favorable environment for presentation and discussion by:* – putting an end to students' autonomous work – providing the reorganization of places to focus on a common resource (whiteboard, overhead…) – promoting an attitude of respect and attentiveness while presentations are being given *Effectively manage relationships among students by:* – establishing the order of presentations – justifying reasons some students do not present (to avoid repetition, …) and changing the order of groups for the next task – promoting and managing student participation in the discussion
Systematizing mathematical learning	*Institutionalize concepts or procedures on mathematical topics by:* – identifying key mathematical concept(s) from the task, clarifying their definition and exploring their multiple representations – identifying key mathematical procedure(s) from the task, clarifying the conditions of their implementation and reviewing how they are used *Institutionalize ideas or procedures concerning the development of transversal skills by:* – identifying and connecting them – enhancing the key factors involved in developing them *Establish connections with prior learning by:* – highlighting links with mathematical concepts, procedures and transversal skills previously explored	*Create an appropriate environment for systematization by:* – focusing students attention on collective systematization – emphasizing the importance of this phase of the lesson for student learning *Guarantee that the systematization ideas are written down by:* – recording them on the computer or other physical devices (boards, interactive boards, transparencies, posters …), which may be done by the students or the teacher – asking students to write down their work

ANNEX I – INTENTIONAL ACTIONS OF THE TEACHER IN INQUIRY-BASED TEACHING (OLIVEIRA, CANAVARRO & MENEZES, 2012A)

projeto
P3M

TASK «ELECTION OF FOR THE CLASS REPRESENTATIVE»

The teacher coordinating the election of the class representative reported that:

- All students in the class voted (30 students) and that there were no null nor blank votes
- Only three students received votes: Francisca, Lucas and Sandra
- Lucas received two votes fewer than Francisca
- Sandra received twice as many votes as Lucas.

Who won the election? With how many votes?

Do not forget to present and explain how you found the solution.

ANNEX 2 – THE MATHEMATICAL TASK (OLIVEIRA, CANAVARRO & MENEZES, 2012A)